D-MAVT
D-MATL

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# Analysis III

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## Serie 6

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<u>**You need to know:**</u> Fourier series, half-range expansion, complex Fourier series.

**1.** Consider the function

$$f(x) = \begin{cases} x, & 0 \leqslant x \leqslant \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} \leqslant x \leqslant \pi \end{cases}$$

**a)** Extend $f$ to an even function on the interval $[-\pi, \pi]$ and then finally to an even, $2\pi$-periodic function on $\mathbb{R}$ and call this function $f_e$. Sketch the graph of $f_e$ and find its Fourier series.

**b)** Do the same for the odd, $2\pi$-periodic extension[1] of $f$ (call this $f_o$).

**2.** Find the Fourier series of the $2L$-periodic extension of

$$f(x) = x, \quad x \in [-L, L)$$

considered in Exercise **5.a)** of Serie 5.

**3.** Find the complex Fourier series of the same function $f(x)$ considered in the previous exercise. Verify that the coefficients $c_n$ of this series

$$\sum_{n=-\infty}^{+\infty} c_n e^{i\frac{n\pi}{L}x}$$

are related as written in the script to the real coefficients $a_n, b_n$ found in the previous exercise.
<u>If you have not computed it before:</u> the real Fourier series of $f$ is

$$\sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2L}{\pi n} \sin\left(\frac{n\pi}{L}x\right) \quad \rightsquigarrow \quad \begin{cases} a_n = 0 \\ b_n = (-1)^{n+1}\frac{2L}{\pi n} \end{cases}$$

[1] To be precise, we can't extend $f$ to an odd, periodic function everywhere because $f(\pi) = \pi$ is not zero. The problematic points are $\pi + 2k\pi, k \in \mathbb{Z}$. Let's assign to these points the value $\pi$.

**Exercises on old topics: Laplace Transform**

**4.** Find the solution $y : [0, \infty) \to \mathbb{R}$ of the following integral equation:

$$y(t) + \int_0^t y(\tau) \cosh(t - \tau)\, d\tau = t + e^t.$$

**5.** Find the inverse Laplace transform of

$$\frac{s}{(s^2 - 16)^2},$$

by using

**a)** the differentiation rule: $\mathcal{L}'(f) = -\mathcal{L}(tf(t))$.

**b)** the integration rule: $\int_s^{+\infty} \mathcal{L}(f)(s')\, ds' = \mathcal{L}\left(\frac{f(t)}{t}\right)(s)$.

**Hand in by:** Thursday 31 October 2019.