D-MAVT
D-MATL

Prof. F. Da Lio
ETH Zürich
Autumn 2019

Analysis III

(stefano.dalesio@math.ethz.ch)

Serie 7

You need to know: convergence of Fourier series, applications to computation of numerical series.

Remark: In the following 'Fourier series' always means the real Fourier series of a function. Otherwise we will always specify 'complex Fourier series'.

1. Let $f(x)$ be the $2L$-periodic extension of x, from the interval $[-L, L)$, as in Exercise **2.** of Serie 6. The Fourier series is

$$f(x) \sim \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2L}{\pi n} \sin\left(\frac{n\pi}{L} x\right).$$

Use this to calculate:

$$\sum_{k=0}^{+\infty} \frac{(-1)^k}{2k+1} = ?$$

2. Let $f(x)$ be the $2L$-periodic extension of x^2 from the interval $[-L, L)$.

a) Compute its Fourier series.

b) The Riemann zeta function is the function

$$\zeta(s) := \sum_{n=1}^{+\infty} \frac{1}{n^s}, \qquad \text{for } s \in \mathbb{C},\ \mathfrak{Re}(s) > 1.$$

Use the previous Fourier series to find the value of the Riemann zeta function in $s = 2$:

$$\zeta(2) = \sum_{n=1}^{+\infty} \frac{1}{n^2} = ?$$

3. For $a > 0$, consider the function $\cosh(ax)$ on the interval $[-\pi, \pi)$ and extend it on all $\mathbb{R}$ to a function of period 2π.

a) Compute its complex Fourier series.

b) Use this result to find the value of the following series:

$$\sum_{n=1}^{+\infty} \frac{(-1)^n}{n^2 + a^2} = ?$$

4. The function $f(x) = \left|\cos\left(\frac{x}{2}\right)\right|$ is periodic of period 2π.

a) Compute its Fourier series.

b) Use this result to find the value of the following series:

$$\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{4n^2 - 1} = ?$$

Hand in by: Thursday 7 November 2019.