Korrekturschema

2. **Emilio+Younghan**

Part (a)
- 3 points are assigned (**3A**) for a correct application of Gaussian elimination to put the matrix in a row echelon form. Every computation mistake costs -1 point; follow-up mistakes do not result in further subtractions of points.
  Notice: for 2 (or more) mistakes in the determination of the matrix of the system, only 1 point is deducted.
  Notice: 1 point is awarded for at least one clearly visible step of Gaussian elimination.
- 2 points (**2B**) are assigned for observing that a necessary condition to have infinitely many solution is that the last row of the reduced matrix vanishes identically, and thus only the values $\lambda = 1$ and $\lambda = 2$ are to be considered;
  Notice: 1 point is deducted for writing that the system has infinitely many solutions when $x_4 = 0/0$.
- 2 points (**2C**) are assigned for excluding the value $\lambda = 1$, which gives a system with no solution.

Part (b)
- 3 points are assigned (**3D**) for correctly determining the values of λ for which A_λ is invertible. Only 1 point is awarded for ruling out just one (correct) value of λ;
- 5 points (**5E**) are assigned for correctly computing the inverse of A_0.
  Notice: in addition to computation mistakes, 1 point is further deducted if the result is a matrix with one vanishing row/column, or with two identical rows/columns (absurd for an invertible matrix).
  Notice: if the student correctly computes the inverse of the reduced form (that is, after Gaussian elimination) of the matrix A_0, instead of the inverse of A_0, only 1 point is awarded.

Notice: for completely unmotivated steps during Gaussian elimination or calculation of the inverse (that is for steps in which no elementary operation on rows/columns is visible), 2 points are deducted.

3. **Tim+Xenia**

(a) **3A** Linearität von F folgt aus drei Fakten (entsprechend drei Punkte):
- Die Ableitung $(-)'$ ist linear.
- Die Multiplikation mit einem Polynom (z.B. $(t+1)$) ist linear.
- Die Summe linearer Abbildungen ist linear.

Die Auflistung genügt hier, es muss jeweils kein Beweis gegeben werden.

3A' Alternativ Linearität von F explizit nachrechnen. Es wird 1 Punkt abgezogen je Rechenfehler/ Ungenauigkeit, sowie falls etwas wichtiges vergessen wird (z.B. nur Additivität, nicht Verträglichkeit mit Skalaren oder umgekehrt).

(b) **5B** Korrekte Berechnung der Bilder einer Basis, sowie Angabe der Matrix. Abzug von 1 Punkt je Rechenfehler. Abzug von 1 Punkt, falls Matrix nicht explizit angegeben.

4B' Korrekte Angabe der Matrix (Rechnung fehlt) mit Angabe bzgl. welcher Basis.

2B' Korrekte Angabe der Matrix (Rechnung fehlt) ohne Angabe bzgl. welcher Basis.

(c) **1C** Charakteristisches Polynom, sowie Angabe der Eigenwerte

3D (je ein Punkt für $p = 2$, $p = 3$, $p > 3$) F diagonalisierbar über allen $\mathbb{F}_p$

3E (je ein Punkt für $p = 2$, $p = 3$, $p > 3$) F invertierbar über $\mathbb{F}_p$ genau dann wenn $p > 3$

4. **Danylo+Tim**

(a) One gets (2A) for considering at the identity $\lambda x_i = \sum_j a_{ij} x_j$, where x_i is the largest entry of a (non-zero) λ-eigenvector v as in the hint. One gets (3B) for applying the triangle inequality to get $|\lambda||x_i| \leqslant \sum_j |a_{ij}||x_j|$ (2B if absolute value forgotten, but otherwise correct. -1P for further mistake/ incomplete argument) and (2C) for showing that this implies $|\lambda| \leqslant \sum_j |a_{ij}|$. Finally, one gets (1D) for concluding using $\sum_j |a_{ij}| \leqslant \max_i \sum_j |a_{ij}|$.

(b) We give (2E) for showing that 23 is an upper bound (1E for incorrect application of (a)) using (a) and (5F) for showing that 23 is an eigenvalue (either by showing that $(1, 1, 1)^T$ is an eigenvector for the eigenvalue 23 or by computing the characteristic polynomial and checking that 23 is its root). In the latter case, we deduct -1P for each mistake/ incomplete argument. One also gets a partial point (1F) for correctly computing the characteristic polynomial ($x^3 - 23x^2 - 208x + 4784$).

5. **Emilio+Danylo**

Part (a) 4 points (**4A**) are assigned for a valid proof of the statement.

Part (b) 4 points (**4B**) are assigned for providing a meaningful counterexample; only 1 point is assigned if the answer is correct, but no valid counterexample is given.

Part (c) 7 points (**7C**) are assigned for a complete proof of the statement.

Notice: no points are awarder neither in (a) nor in (c) for the proof that similar matrices have same characteristic polynomial, which was seen in the lecture (and in (c) it is not even the right path to the solution).

6. **Danylo+Xenia**

(a) If a proof uses Sylvester's theorem $\det(E_n + vu^T) = 1 + v^T u$: (3A) for the idea of using Sylvester's theorem, (3B) for noting that it applies to A_n by writing it in the form $(a-1)E_n + \mathbf{1}\mathbf{1}^T$ or its equivalent, and (3C) if the calculation of $\det(A_n)$ using Sylvester's theorem is done without mistakes.

If a proof uses induction: (2A) for the idea of using induction and checking the base case $n = 1$ (or $n = 1, 2$). (4B) is given for proving a 3-term identity involving $\det(A_n)$, $\det(A_{n-1})$, and $\det(B_n)$ as in the official solution, or an equivalent formula. Simply applying Laplace expansion as an attempt to prove the inductive step is worth (1B). Finally, (3C) is given for the correct evaluation of $\det(B_n)$, or otherwise correct proof of the inductive step.

For solutions that directly use row/column transformations: we give (3A) for the idea of using row transformations to put a matrix into upper/lower triangular form, (5B) for a successful implementation of this idea, and (1C) for reading off the determinant of a transformed matrix.

(b) One gets (3D) for correctly calculating $\det(B)$, (2E) for correctly showing that $z = -b$, only (1E) if one does not rule out $z = 0$, and (1F) for showing that $z = -b$ implies $b^2 - bc + 1 = 0$.