Complex Analysis Exercise 11

Prof. Dr. Paul Biran

Due: 29.11.2019

1. Compute Laurent series for the following functions:

   (a) $\frac{e^{1-z}}{z^2}$ around 0

   (b) $z^4(e^{\frac{1}{z^2}} - 1)$ around 0

   (c) $e^{z+\frac{1}{z}}$ in the annulus $0 < |z| < \infty$

   (d) $\frac{1}{1-z^2}$ in every annulus around 0 and around 1.

2. Let $f(z) = \frac{z-7}{z^2+z-2}$. Find Laurent expansion for every relevant annulus around zero.

3. (a) Let $f \in \text{Hol}(|z| > 1)$. Assume that f satisfies $\int_{|z|=2} f(z)dz = 0$. Show that f has a primitive in $\{|z| > 1\}$.
(b) Let $f \in \text{Hol}(|z| > R)$ for some $R > 0$ and $f(-z) = f(z)$. Prove that f has a primitive in $\{|z| > R\}$ and $\int_{|z|=r} f dz = 0$, $r > R$.

4. Prove the following statement: Let f be holomorphic in $a < |z| < b$. Then for every $a < r_1 < r_2 < b$,

$$\int_{|z|=r_1} f dz = \int_{|z|=r_2} f dz.$$

5. Find all $\lambda \in \mathbb{C}$ for which there exists entire function f such that $\forall z$, $f(z) = f(\lambda z)$ (f is not constant).