Complex Analysis Exercise 2

Prof. Dr. Paul Biran

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1. Let A be an open and path-connected subset in $\mathbb{C}$. If we write

$$A = U \cup V$$

where U and V are open subsets and $U \cap V = \emptyset$, then prove either $U = A$, $V = \emptyset$ or $U = \emptyset$, $V = A$.

2. (a) Prove (without using the Cauchy-Riemann equation) that functions

$$f(z) = Re(z),\ g(z) = Im(z)$$

are not differentiable at any point.
(b) Let $a, b \in \mathbb{C}$. Find all points in $\mathbb{C}$ where $af(z) + bg(z)$ is differentiable.

3. Find at which points derivatives of the following functions exists. Compute these derivatives.

(a) $\frac{3z^2+2z}{z^4-1}$

(b) $e^{\overline{z}}$

(c) $z(z + \overline{z}^2)$.

4. Prove

(a) $cos(\frac{\pi}{2} - z) = sin(z)$

(b) $cos(z) = cosh(iz)$.