

Let X be a topological space and $f: X \rightarrow X$ be a continuous map. The *mapping torus* is the topological space $T_f = X \times [0, 1]/(x, 0) \sim (f(x), 1)$. There is a long exact homology sequence

$$\cdots \rightarrow H_n(X) \xrightarrow{\text{Id}-f_*} H_n(X) \rightarrow H_n(T_f) \rightarrow H_{n-1}(X) \rightarrow \cdots$$

Question 1:

Compute the homology of the mapping torus associated to a reflection $f: S^2 \rightarrow S^2$.

Question 2:

Compute the homology of the mapping torus of a map $S^1 \times S^1 \rightarrow S^1 \times S^1$ that is the identity on one factor and a reflection on the other.

Question 3:

For ΣX the suspension of X , show by a Mayer-Vietoris sequence that there are isomorphisms $\tilde{H}_n(\Sigma X) \rightarrow \tilde{H}_{n-1}(X)$ for all n .