Algebraic Topology **Exercise Sheet 3**

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Let X, Y be topological spaces and $f\colon X \to Y$ be a continuous map. We will use the notation $f_{\#}\colon \pi_1(X, x) \to \pi_1(Y, f(x))$ to denote the induced map on the level of fundamental groups and $f_*\colon H_n(X) \to H_n(Y)$ the induced map on the level of homology.

Question 1:

[Problem 2, Page 177, Bredon] If $f\colon X \to Y$ is a map and $f(x_0) = y_0$ show that the diagram

$$\begin{tikzcd} \pi_1(X, x_0) \arrow[r, "f_{\#}"] \arrow[d, "\phi_X"] & \pi_1(Y, y_0) \arrow[d, "\phi_Y"] \\ H_1(X) \arrow[r, "f_*"] & H_1(Y) \end{tikzcd}$$

commutes, where ϕ_X and ϕ_Y are the Hurewicz homomorphisms.

Question 2:

[Problem 3, Page 177, Bredon] If $f\colon X \to Y$ is a covering map then $f_{\#}$ is an injective homomorphism by covering space theory. Is it true that $f_*\colon H_1(X) \to H_1(Y)$ is also injective? Give either a proof or a counterexample.

Question 3:

[Problem 1, Page 182, Bredon] Multiplication by a non-zero integer $n\colon \mathbb{Z} \to \mathbb{Z}$ fits in a short exact sequence

$$0 \to \mathbb{Z} \xrightarrow{p} \mathbb{Z} \to \mathbb{Z}/p\mathbb{Z} \to 0.$$

Use this to derive the exact sequence

$$0 \to \frac{H_n(X)}{pH_n(X)} \to H_n(X; \mathbb{Z}/p\mathbb{Z}) \to \ker\{p\colon H_{n-1}(X) \to H_{n-1}(X)\} \to 0.$$