Algebraic Topology **Exercise Sheet 6**

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Some notes on definitions:

1. With the notation $X \sqcup Y$, we mean the topological space where the sets X, Y are clopen, and the inclusions $X \to X \sqcup Y$, $Y \to X \sqcup Y$ are homeomorphisms on their images.
2. Given pointed spaces (X_i, p_i), we denote $\vee X_i = \bigsqcup X_i / p_i \sim p_j$.
3. Given a topological space X and a subset $A \subseteq X$, we define $X/A = X \sqcup \{*\}/y \sim x \forall x, y \in A \cup \{*\}$.

Question 1:

Compute $\tilde{H}(\vee_{i \in I} S^n_i)$, where S^n is the n–dimensional sphere.

Question 2:

Let $f_\sigma \colon D^n_\sigma \to K^{(n)}$ be the attaching map of an n–cell, let $S^n_\sigma = K^{(n)}/K^{(n)} - f_\sigma(\mathring{D}^n_\sigma)$, and let $p_\sigma \colon K^{(n)} \to S^n_\sigma$ be the quotient map.

1. Show that $(p_\sigma \circ f_\sigma)_* \colon H_*(D^n_\sigma, \partial D^n_\sigma) \to H_*(S^n_\sigma, \{*\})$ is an isomorphism.
2. Show that for $\tau \neq \sigma$, the map $(p_\sigma \circ f_\tau)_*$ is the zero map.

Question 3:

Let X be a topological space. Show that there is an isomorphism $\tilde{H}_*(X) \to H_*(X, \{*\})$ which factors through the inclusion $X \to (X, \{*\})$.