Algebraic Topology

Prof. Dr. Alessandro Sisto Assistant: Davide Spriano

Question 1:

[Hatcher Ex 1 p. 155] Construct a surjective map $S^n \to S^n$ of degree zero, for each $n \ge 1$.

Question 2:

[Hatcher Ex 9 p. 156] Compute the homology of the following 2-complexes:

- 1. The quotient of S^2 obtained identifying north and south poles to a point.
- 2. The space obtained from D^2 by first deleting the interiors of two disjoint subdisks in the interior of D2 and then identifying all three resulting boundary circles together via homeomorphisms preserving clockwise orientations of these circles.

Question 3:

[Hatcher Ex 12 p. 156] Show that the quotient map $S^1 \times S^1 \to S^2$ obtained by collapsing the subspace $S^1 \vee S^1$ to a point is not null homotopic (i.e. homotopic to a constant map) by showing that it induces an isomorphism on H_2 .

Bonus: On the other hand, show via covering spaces that any map $S^2 \to S^1 \times S^1$ is nullhomotopic.