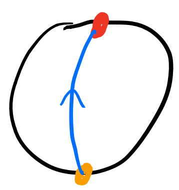
Q2) 1) CW STRUCTURE FOR $S^2/N\sim S$

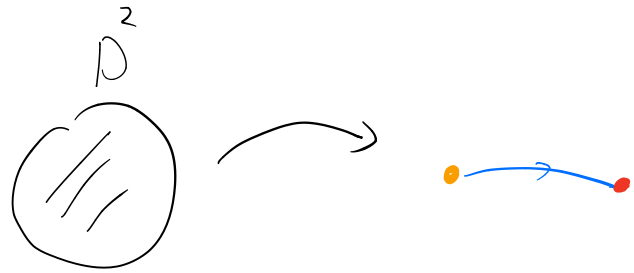
START WITH

- CW STRUCTURE FOR S^2 w/ different 0-cells for $N \neq S$.

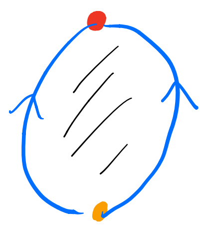
ONE 2-cell, ONE 1-cell, TWO 0-cells.



i.e.

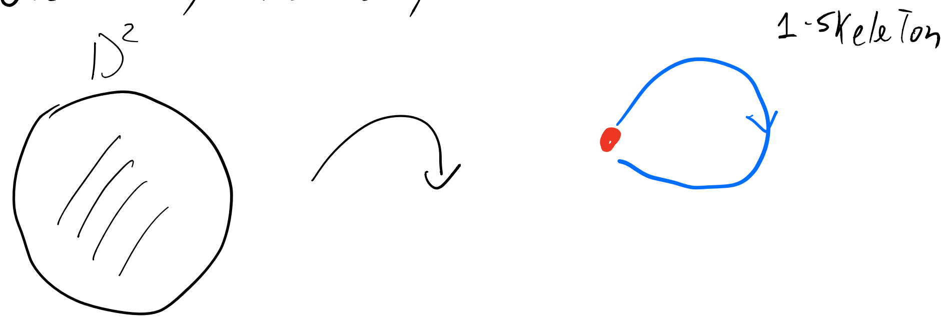


via



AFTER QUOTIENT, we have THAT $S^2/N\sim S$ HAS THE FOLLOWING CW STRUCTURE:

ONE 2-cell; ONE 1-cell; ONE 0-cell.



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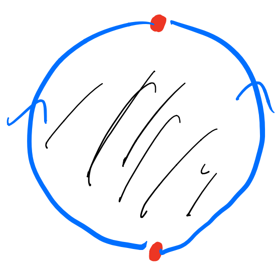


THE DEGREE OF THE MAP $\partial D^2 \xrightarrow{\phi} 1\text{-skeleton}$

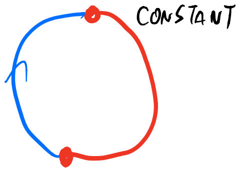
is 0.

Complement of interior of 1-cell

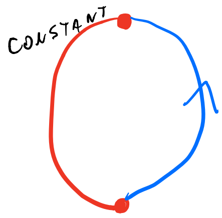
in this particular example, we are quotienting by a set containing exactly one point. So NOTHING HAPPENS.

INDEED, $\deg \phi = \deg \phi_1 + \deg \phi_2$,

where

$\phi_1 =$ 

and

$\phi_2 =$ 

Since $\phi_2 = \phi_1 \circ \big((x,y) \longmapsto (-x,y) \big) \Rightarrow \deg(\phi_1) = -\deg(\phi_2)$

Thus we need to compute the homology of the chain cpx:

$$0 \to \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \to 0$$

$$\rightsquigarrow \quad H_n(S^2/_{N\sim S}) = \begin{cases} \mathbb{Z} & , n = 0,1,2 \\ 0 & \text{———} \end{cases}$$

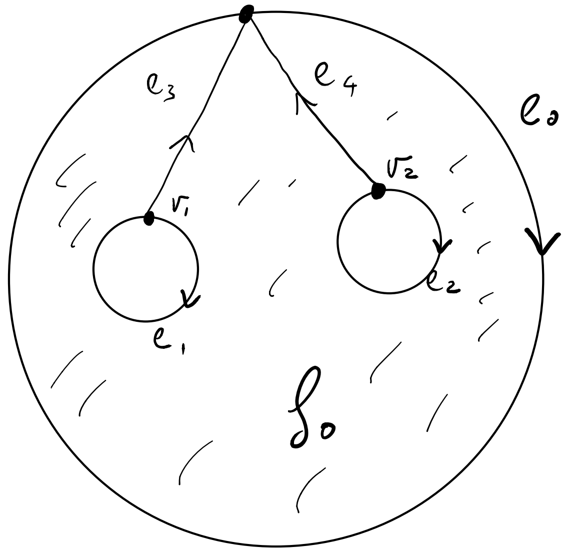
Q2 | PART 2)

Let X be the space described in the exercise.

Then X is the QUOTIENT OF THE FOLLOWING SPACE:

v_0

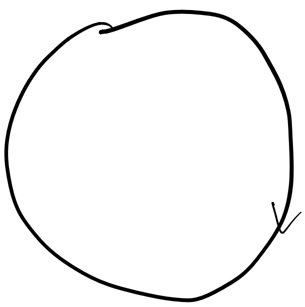
Y =



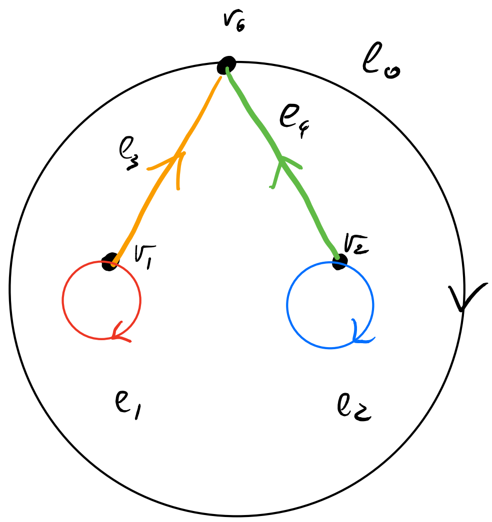
Y has THREE 0-cells (v_0, v_1, v_2);
FIVE 1-cells $(e_0, \dots, e_4)$
and ONE 2-cell. (f_0)

THE ATTACHING MAP OF f_0

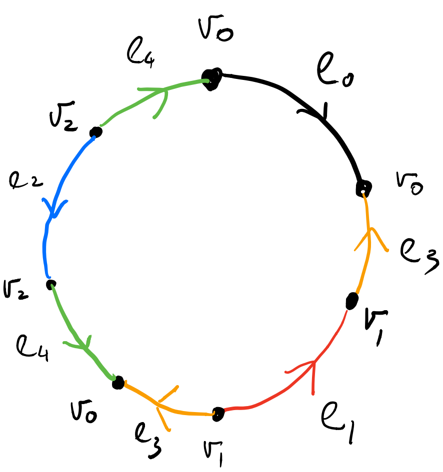
∂D_2



$\to$



is



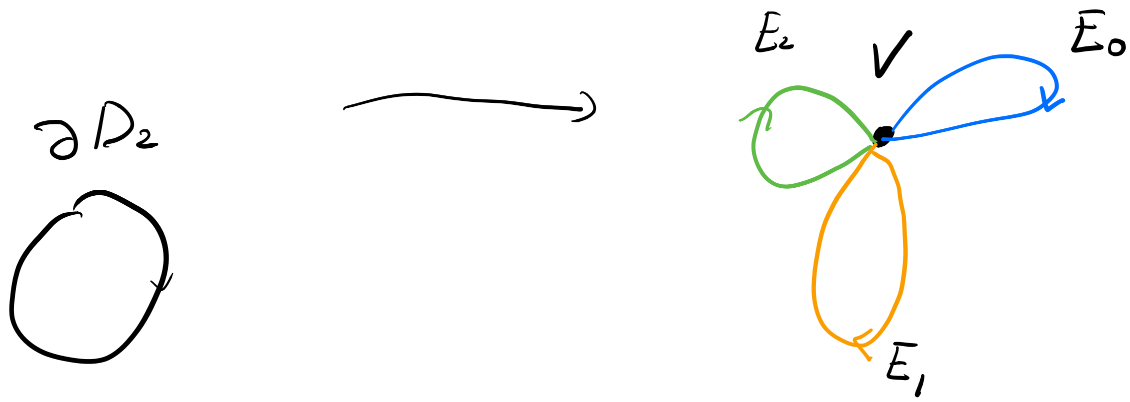
X is obtained from Y by identifying the 0-cells together, and e_0, e_1, e_2 respecting the orientation.

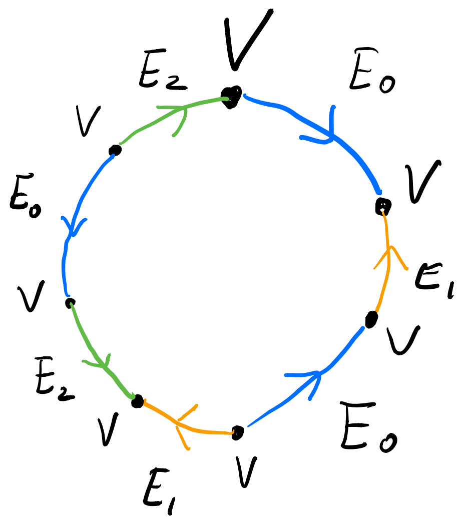
Thus, X has: one 0-cell $(V = [v_0, v_1, v_2])$

three 1-cells $\left(E_0 = [e_0, e_1, e_2],\ E_1 = [e_3],\ E_2 = [e_4] \right)$

one 2-cell $(F = [f_0]$

The attaching map of F is





(i.e. The quotient of the map above)

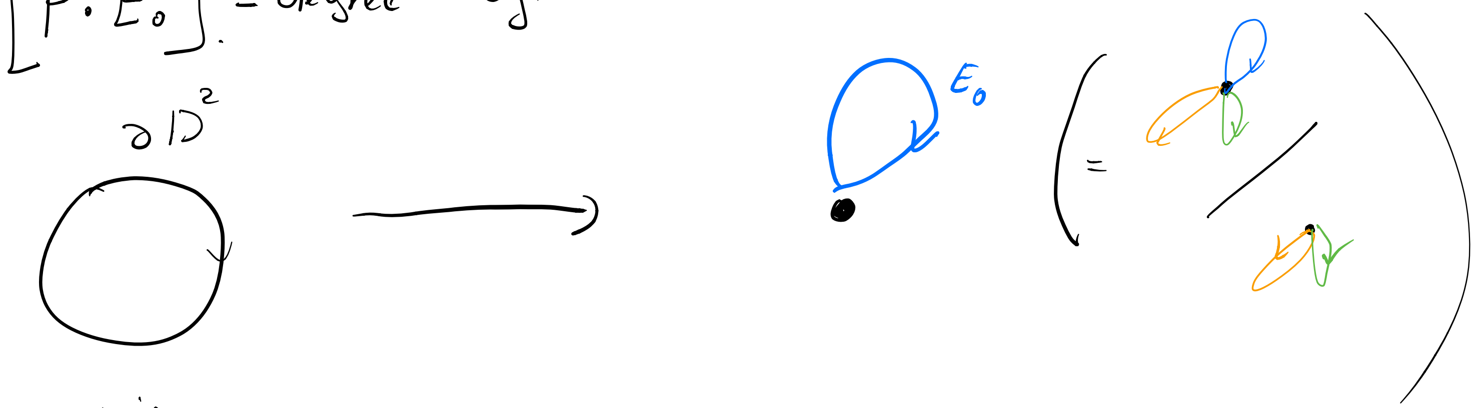
To compute the homology of X, we need to compute the homology of the complex

$$0 \to \mathbb{Z} \xrightarrow{\beta_2} \mathbb{Z}^3 \xrightarrow{\beta_1} \mathbb{Z} \to 0.$$

The map β_1 is the ZERO MAP BECAUSE THERE IS ONLY ONE 0-CELL.

β_2 is GIVEN BY $\begin{pmatrix}[F:E_0]\\ [F:E_1]\\ [F:E_2]\end{pmatrix}$.

$[F:E_0]$ = degree of:



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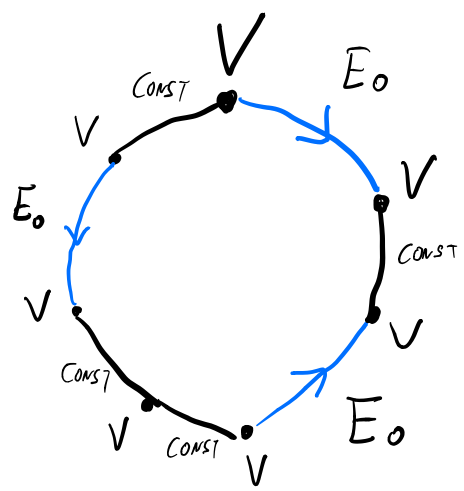
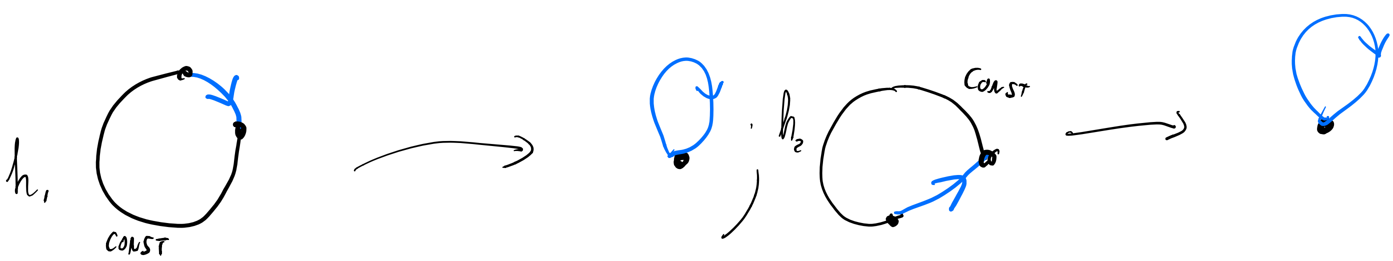
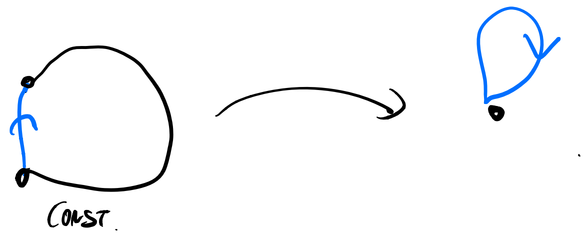


AS USUAL, TO COMPUTE THE DEGREE OF THE ABOVE MAP IT SUFFICES TO COMPUTE THE DEGREES OF THE MAPS:



h_3



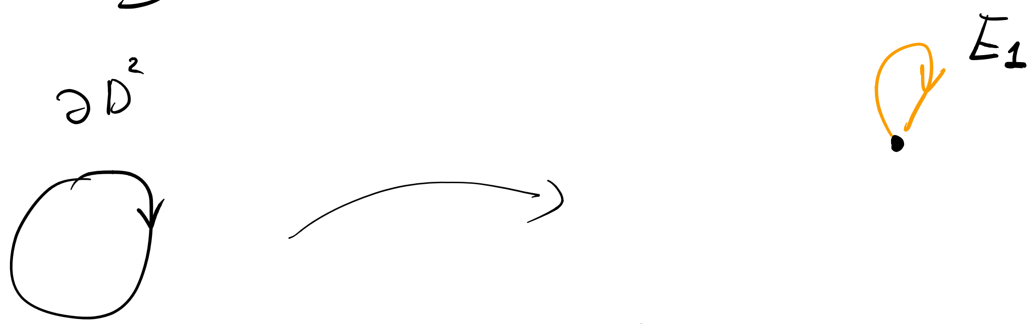
OBSERVE THAT h_1 IS HOMOTOPIC TO THE IDENTITY, AND h_2, h_3 are obtained FROM h_1 BY FLIPPING ONE COORDINATE.

<u>THUS:</u>

$$[F : E_0] = \deg(h_1) + \deg(h_2) + \deg(h_3) = 1 - 1 - 1$$

$$= -1.$$

$[F:E_1]$ = degree of



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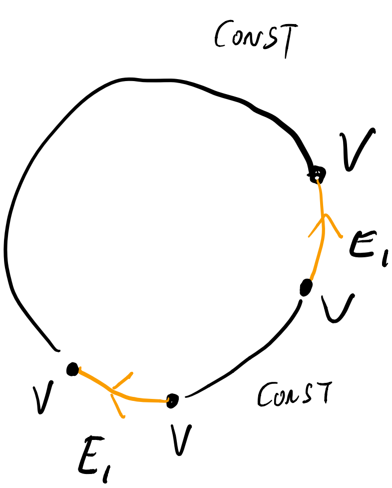

REASONING AS BEFORE WE HAVE

$[F:E_1]=0.$

COMPUTING $[F:E_2]$ is TOTALLY analogous To $[F:E_1]$. Thus

$[F:E_2]=0.$

We HAVE:

$$0 \to \mathbb{Z} \to \mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z} \xrightarrow{0} \mathbb{Z} \to 0$$

$$(1) \quad 1 \longmapsto (-1,0,0)$$

The homology OF THE CHAIN Complex is:

$H_0(X)= \mathbb{Z}$

$H_1(X)= \dfrac{\mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}}{\mathbb{Z}\oplus\{0\}\oplus\{0\}} = \mathbb{Z}\oplus\mathbb{Z}$

$H_2(X)= \{0\}/\{0\} = \{0\}$