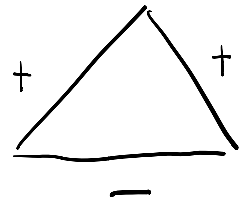
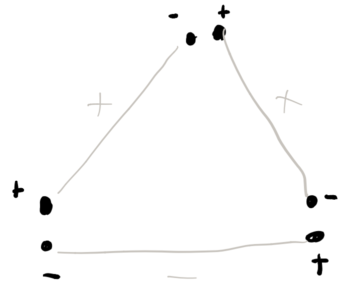
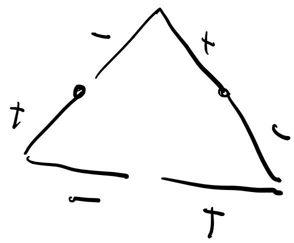
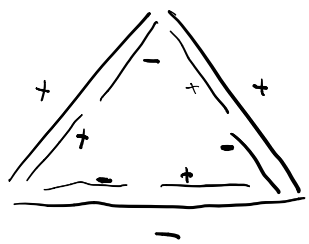
Q 1 | Easier version.

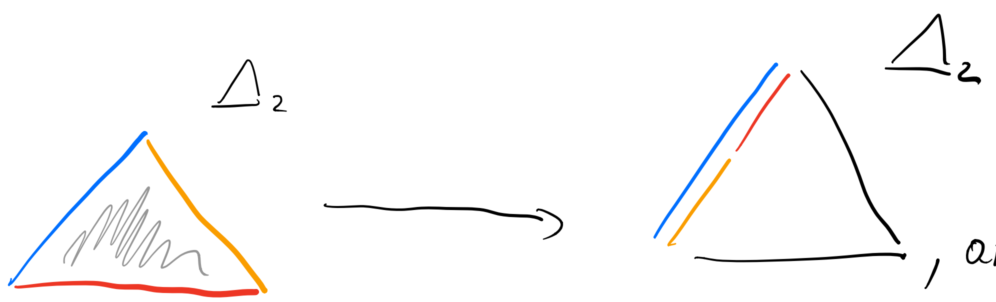
$a =$  , $\partial a =$ 

$b =$ Baricentric $(a) =$ 

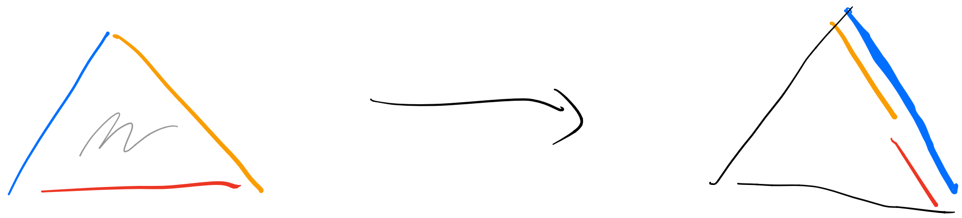
$a - b =$ 

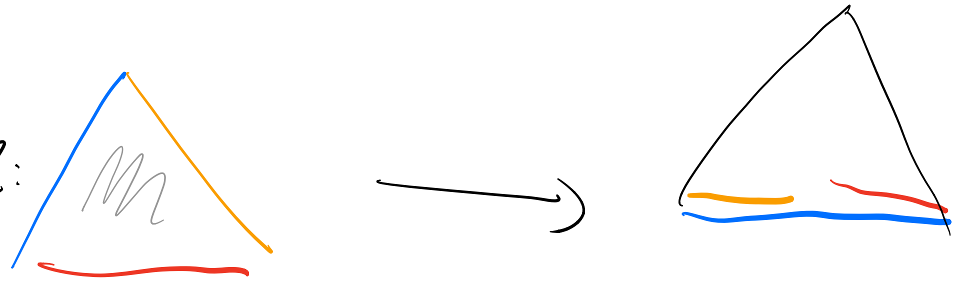
Note $a - b \in \Delta_1(\partial \Delta^2)$. We will now find a chain $c \in \Delta_2(\Delta^2)$ such that $\partial c = (a - b)$

Let $X \in \Delta_2(\Delta^2)$ be the singular 2-simplex

$X:$ 

, and similarly

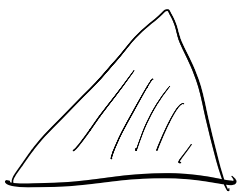
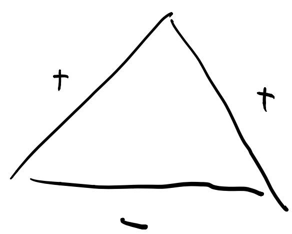
Y: 

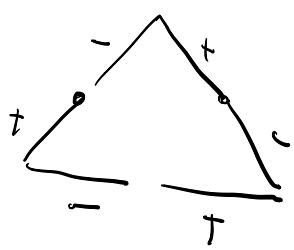
Z: 

Then $c = X + Y - Z$ and

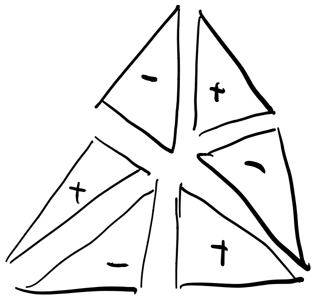
$$\partial c = (a - b)$$

Harder version:

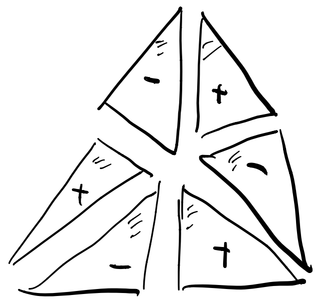
$a =$  , $\partial a =$  ;

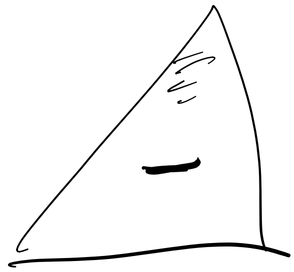
BARICENTRIC $(\partial a) =$ 

$b = $ BARICENTRIC $(a) =$



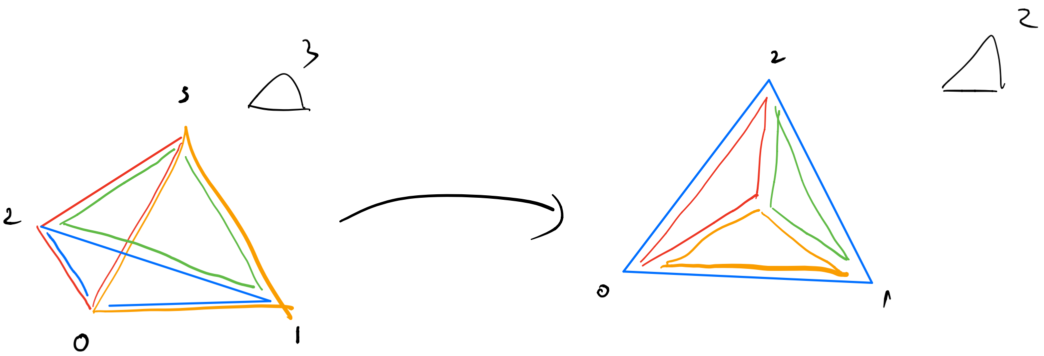
$b - a =$



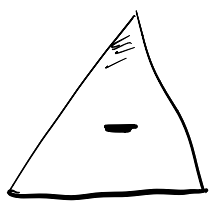


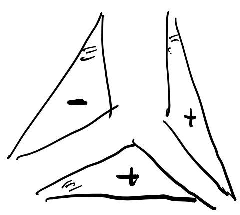
let $X: \Delta^3 \longrightarrow \Delta^2$ be

$X:$

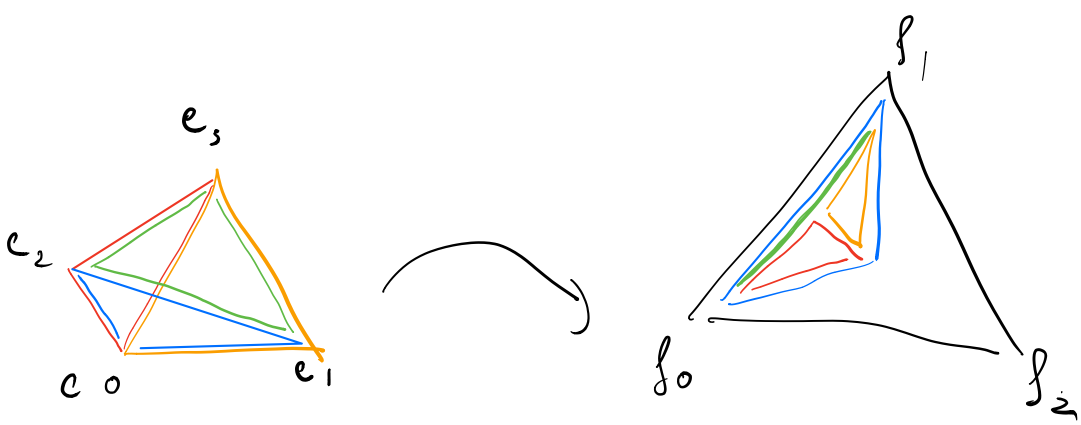


Then, $\partial X =$





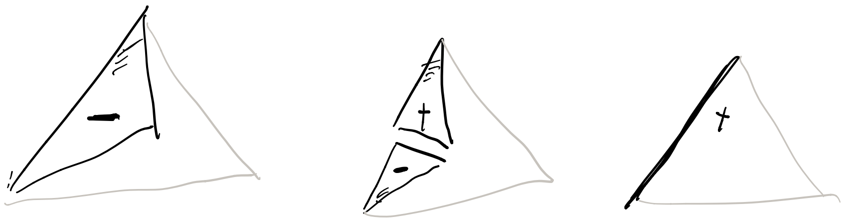
$Y:$



IN FORMULAS:

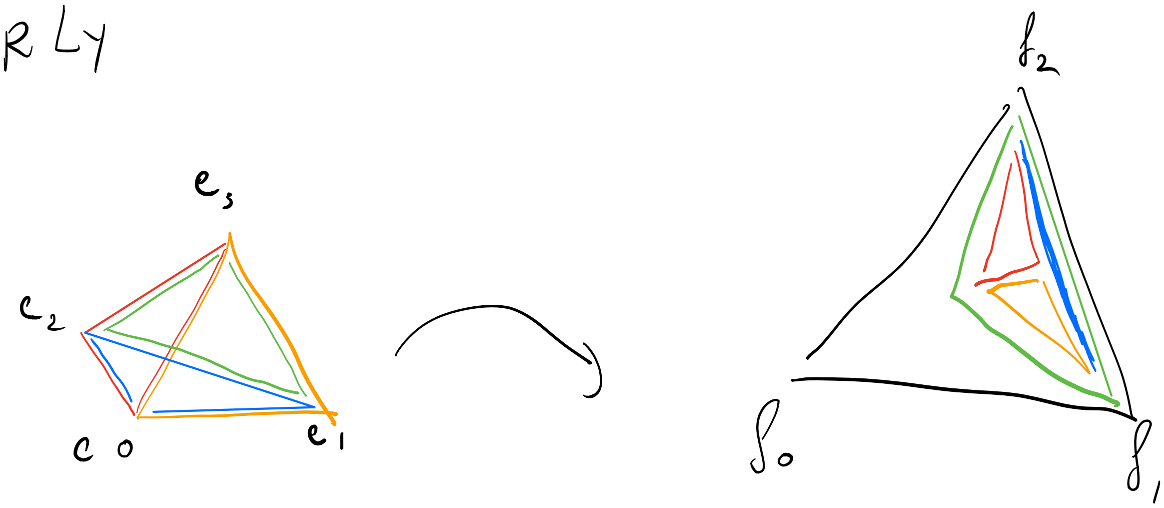
$$\lambda_0 e_0 + \lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3 \longmapsto \left(\tfrac{1}{3}\lambda_0 + \lambda_2 + \tfrac{1}{2}\lambda_3\right) f_0 + \left(\tfrac{1}{3}\lambda_0 + \lambda_1 + \tfrac{1}{2}\lambda_3\right) f_1 + \left(\tfrac{1}{3}\lambda_0\right) f_2$$

$\partial Y =$

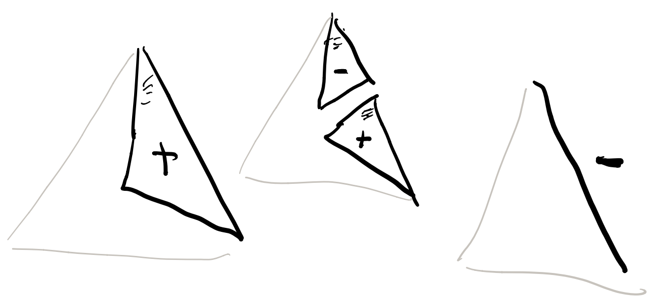


SIMILARLY

$Z =$

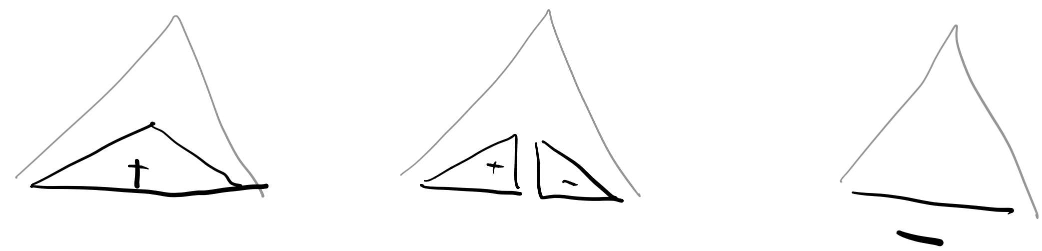


$\partial Z =$

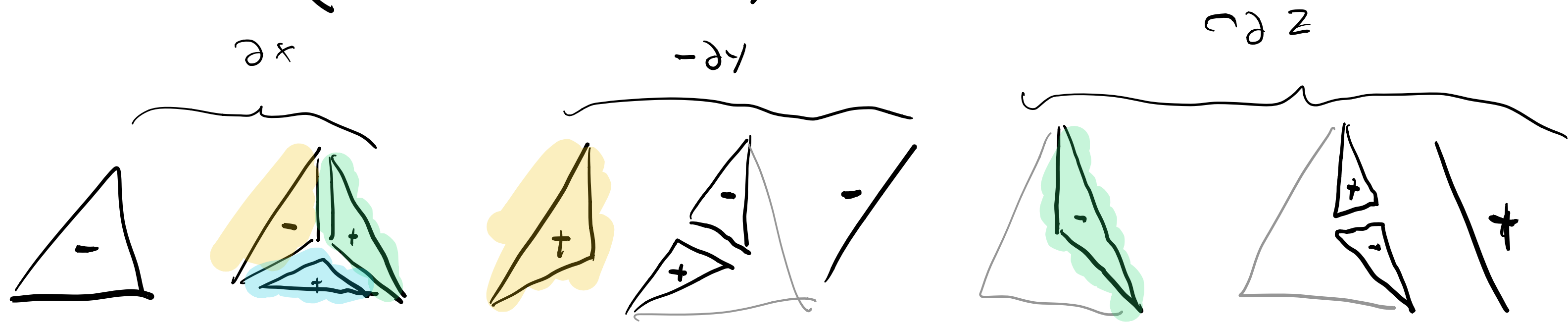


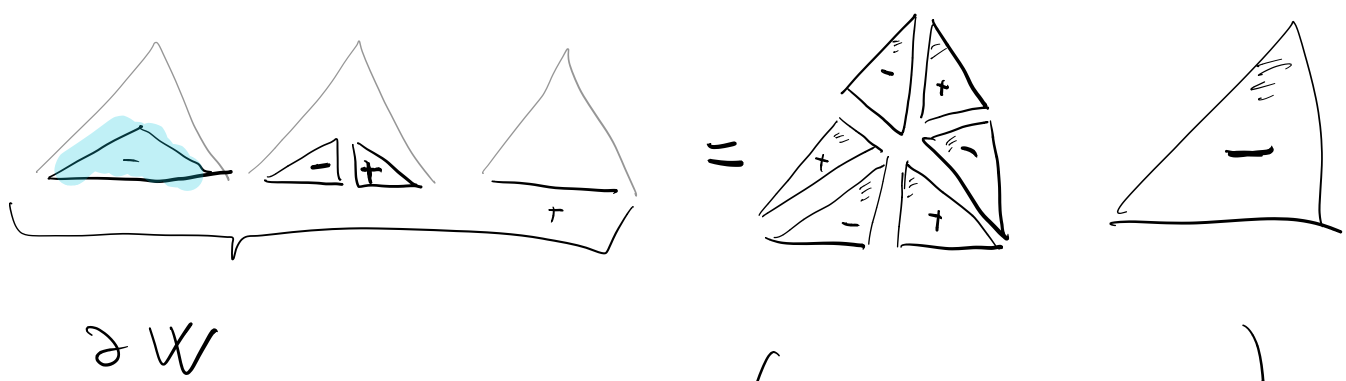
$W =$

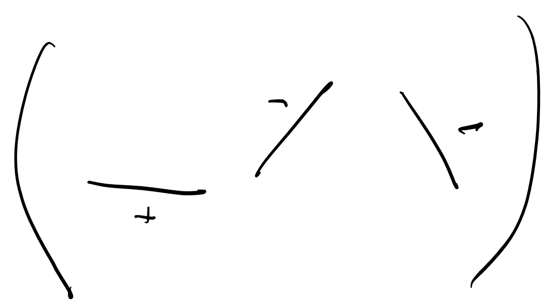
$\partial W =$



$\boxed{50}$ $\partial(X - Y - Z - W) =$







if $c = X - Y - Z - W$, we have

$$\partial c = a - b + d, \quad d \in \Delta_2(\partial \Delta^2).$$