Algebraic Topology **Mock Exam**

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Question 1:

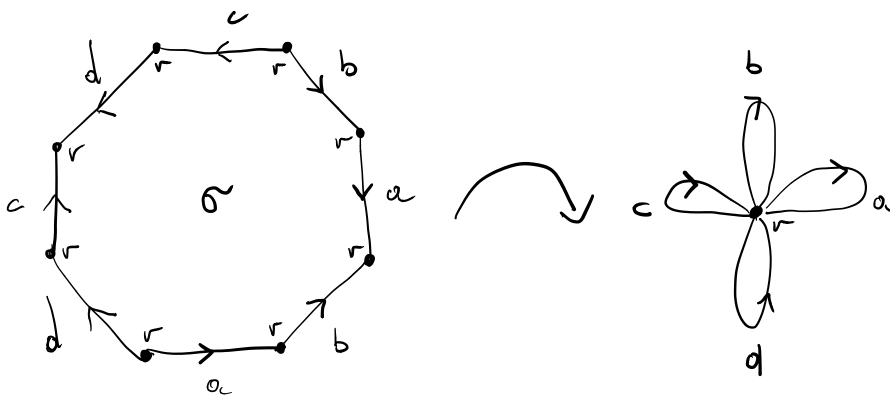
1. State the Hurewicz Theorem about the connection between homology and fundamental group.
2. The proof of Hurewicz Theorem uses a certain map $\Psi\colon \Delta_1(X) \to \pi_1^{\mathrm{ab}}(X)$. Define the map.
3. Show that $\Psi(B_1(X)) = 0$.

Question 2:

1. State the excision Theorem for singular homology.
2. Define the map $\Upsilon\colon L_p(\Delta^q) \to L_p(\Delta^q)$, where $L_p(\Delta^q)$ denotes the affine singular simplices of Δ^q.
3. Proove that Υ is a chain map.

Question 3:

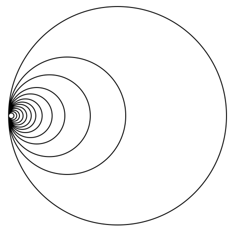
Let Σ_2 be the CW-complex with one 0-cell labelled v, four 1-cells labelled a, b, c, d and one 2-cell labelled σ with attaching map as in the following picture.



1. Compute the singular homology groups $H_*(\Sigma_2)$ of Σ_2.
2. Let X be a finite CW-complex such that $X^{(2)}$ is homeomorphic to Σ_2. What can you say about $H_1(X)$?
3. What can you say about $H_2(X)$?

Question 4:

For each integer $n \geq 1$, let C_n be the circle of radius $\frac{1}{n}$ centered in $\left(\frac{1}{n}, 0\right)$. For clarity, $C_n = \left\{(x, y) \mid \left(x - \frac{1}{n}\right)^2 + y^2 = \frac{1}{n^2}\right\}$. Let $X = \bigcup_{n=1}^{\infty} C_n \subseteq \mathbb{R}^2$ be endowed with the subspace topology.



1. Show that the singular homology group $H_1(X)$ is not finitely generated. *Possible hint: using suitable maps to the various circles may help*
2. For each $n > 0$, let $i_n \colon C_n \to X$ be the inclusion. Show that the homorphism

$$\bigoplus (i_n)_* \colon \bigoplus H_1(C_n) \to H_1(X)$$

is not surjective. That is to say that $H_1(X)$ is not generated by the images of the first homology groups of the circles C_n.