

# Non-Life Insurance: Mathematics and Statistics

## Exercise sheet 4

### Exercise 4.1 Poisson Model and Negative-Binomial Model

Suppose that we are given the following claim count data of ten years:

| $t$   | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 10     |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $N_t$ | 1'000  | 997    | 985    | 989    | 1'056  | 1'070  | 994    | 986    | 1'093  | 1'054  |
| $v_t$ | 10'000 | 10'000 | 10'000 | 10'000 | 10'000 | 10'000 | 10'000 | 10'000 | 10'000 | 10'000 |

Table 1: Observed claim counts  $N_t$  and corresponding volumes  $v_t$ .

- Estimate the claim frequency parameter  $\lambda > 0$  of the Poisson model. Moreover, calculate a prediction interval which should contain roughly 70% of the observed claim frequencies  $N_t/v_t$ . What do you observe?
- Perform a  $\chi^2$ -goodness-of-fit test at significance level of 5% to test the null hypothesis of having Poisson distributions.
- Estimate the claim frequency parameter  $\lambda > 0$  and the dispersion parameter  $\gamma > 0$  of the negative-binomial model. Moreover, calculate a prediction interval which should contain roughly 70% of the observed claim frequencies  $N_t/v_t$ . What do you observe?

### Exercise 4.2 $\chi^2$ -Goodness-of-Fit-Analysis (R Exercise)

In this exercise we analyze the sensitivity of the  $\chi^2$ -goodness-of-fit test (of having a Poisson distribution as claim count distribution) in situations where the claim counts are simulated from a Poisson distribution and a negative binomial distribution, respectively.

- Write an R code that generates  $n = 10'000$  times claim counts  $N_1, \dots, N_T \stackrel{\text{i.i.d.}}{\sim} \text{Poi}(\lambda v)$  with  $T = 10$ ,  $\lambda = 10\%$  and  $v = 10'000$ . Apply for each of these  $n$  replications of  $N_1, \dots, N_T$  a  $\chi^2$ -goodness-of-fit test at significance level of 5% of having a Poisson distribution as claim count distribution. Answer the following questions:
  - What can you say about the distribution of the  $n$  MLEs of  $\lambda$ ?
  - Consider a QQ plot to analyze whether the  $n$  values of the test statistic may indeed come from a  $\chi^2$ -distribution with  $T - 1 = 9$  degrees of freedom.
  - How often do we wrongly reject the null hypothesis  $H_0$  of having a Poisson distribution as claim count distribution?
- Write an R code that generates  $n = 10'000$  times claim counts  $N_1, \dots, N_T \stackrel{\text{i.i.d.}}{\sim} \text{NegBin}(\lambda v, \gamma)$  with  $T = 10$ ,  $\lambda = 10\%$ ,  $v = 10'000$  and  $\gamma \in \{100, 1'000, 10'000\}$ . Apply for each of these  $n$  replications of  $N_1, \dots, N_T$  a  $\chi^2$ -goodness-of-fit test at significance level of 5% of having a Poisson distribution as claim count distribution. Answer the following questions:
  - How often are we able to reject the null hypothesis  $H_0$  of having a Poisson distribution as claim count distribution?
  - Does the size of  $\gamma$  influence this percentage?

**Exercise 4.3 Claim Count Distribution**

Suppose that in a given line of business of an insurance company the numbers of claims of the last ten years are modeled by random variables  $N_1, \dots, N_{10}$ . We assume that  $N_1, \dots, N_{10}$  are i.i.d. and that we have collected the following observations:

|       |   |    |    |    |    |    |    |   |    |    |
|-------|---|----|----|----|----|----|----|---|----|----|
| $t$   | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8 | 9  | 10 |
| $N_t$ | 7 | 21 | 19 | 18 | 25 | 17 | 33 | 6 | 39 | 28 |

Table 2: Observed numbers of claims  $N_t$  over the last ten years.

Which claim count distribution would you prefer in this situation? Give a short argument.

**Exercise 4.4 Method of Moments**

The i.i.d. claim sizes  $Y_1, \dots, Y_8$  are supposed to follow a Gamma distribution with unknown shape parameter  $\gamma > 0$  and unknown scale parameter  $c > 0$ . We assume that we have the following observations for  $Y_1, \dots, Y_8$ :

$$y_1 = 7, \quad y_2 = 8, \quad y_3 = 10, \quad y_4 = 9, \quad y_5 = 5, \quad y_6 = 11, \quad y_7 = 6, \quad y_8 = 8.$$

Use the method of moments to estimate the parameters  $\gamma$  and  $c$ .