Mathematical Finance

Exercise sheet 12

Exercise 12.1 (optional) Consider the utility function $u_\gamma(x) = \frac{x^\gamma}{\gamma}$, for $x > 0$ and $\gamma \in (-\infty, 1)\setminus\{0\}$. Show that $u_\gamma(x) - \frac{1}{\gamma} \to \log x$ as $\gamma \to 0$. Compute the conjugate functions of u_γ and log.

Exercise 12.2 Assume that the interest rate is 0, i.e. there exists a riskless asset with constant value 1, and consider the Bachelier model

$$dS_t = \mu dt + \sigma dB_t, \quad S_0 \in \mathbb{R},$$

with $\mu \in \mathbb{R}$ and $\sigma > 0$.

Compute the optimal utility and optimal strategy associated with the problem

$$J_0 = \sup_{\vartheta \in \Theta^x_{\text{adm}}} E\left[u\left(x + \int_0^T \vartheta_s dS_s\right)\right],$$

for the cases of power utility $u_\gamma(x)$ and log-utility $u(x) = \log(x)$.

Hint. To find a good ansatz for the log-utility case, try (heuristically) taking a limit of the power utility case as $\gamma \to 0$.