

Mathematical Finance

Exercise sheet 6

Exercise 6.1 Show that in finite discrete time we have

$$(NA) \implies (NUPBR).$$

Exercise 6.2 Let X be a Banach space and let Y be a closed point-separating linear subset of the dual space. Show that X , together with the topology making all linear functionals of Y continuous (the $\sigma(X, Y)$ topology) is metrizable if and only if X is finite dimensional.

Exercise 6.3 Let $C \subseteq X$ be a convex subset of X , a Banach space. Show that C is closed in X if and only if it is closed with respect to the weak topology $\sigma(X, X^*)$.

Exercise 6.4 (Python) Let B be a standard Brownian motion and consider a market consisting of three assets $S^0 \equiv 1$, $S_t^1 = \exp(B_t)$ and $S_t^2 = \exp(\frac{1}{2}B_t)$, $t \in [0, T]$, for some $0 < T < \infty$. Verify numerically that the market admits scalable arbitrage.