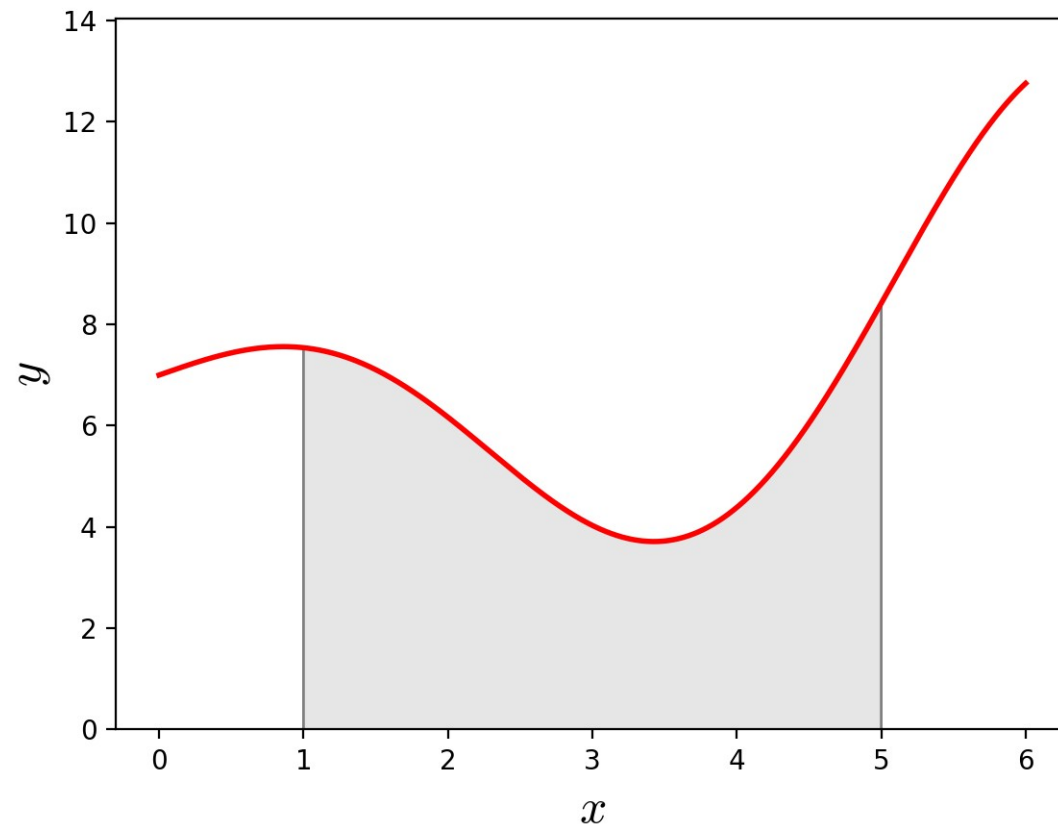


Bsp.: (9)

Quadratur-Regeln

$$f(x) = x \cos(x) + 7$$

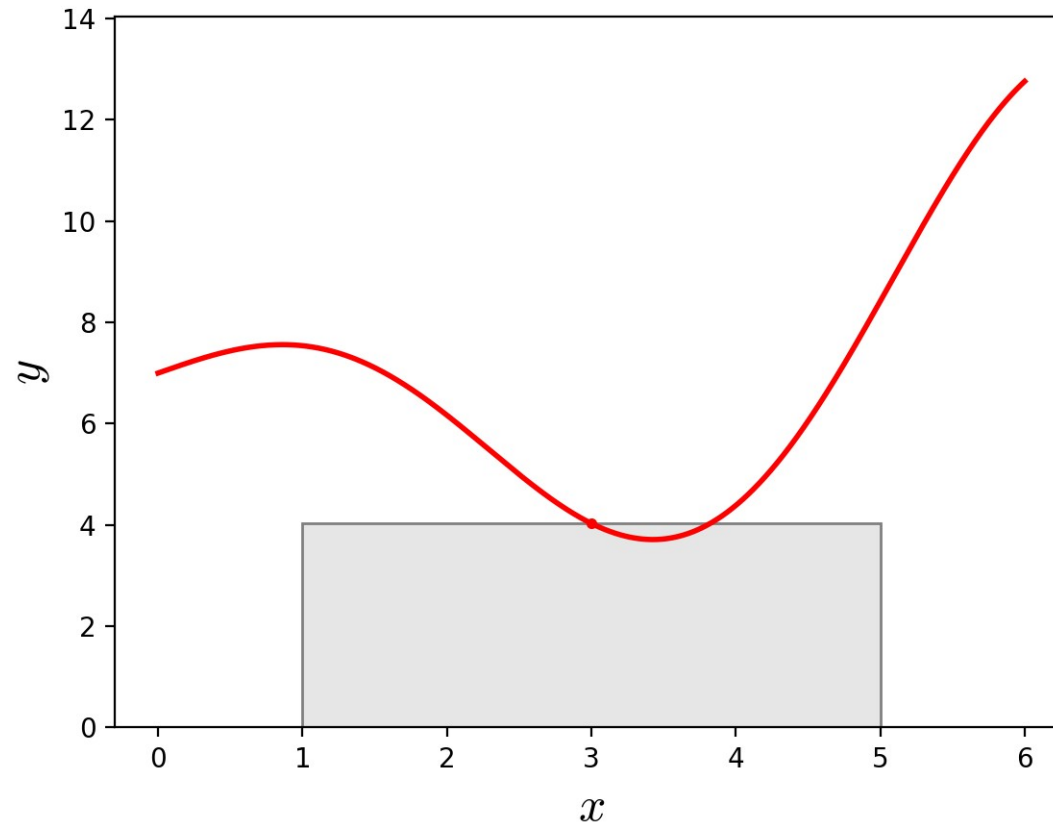


$$I[f] = \int_1^5 f(x) dx = 22.1073\dots$$

Bsp.: (9)

Mittelpunktsregel

$$f(x) = x \cos(x) + 7$$

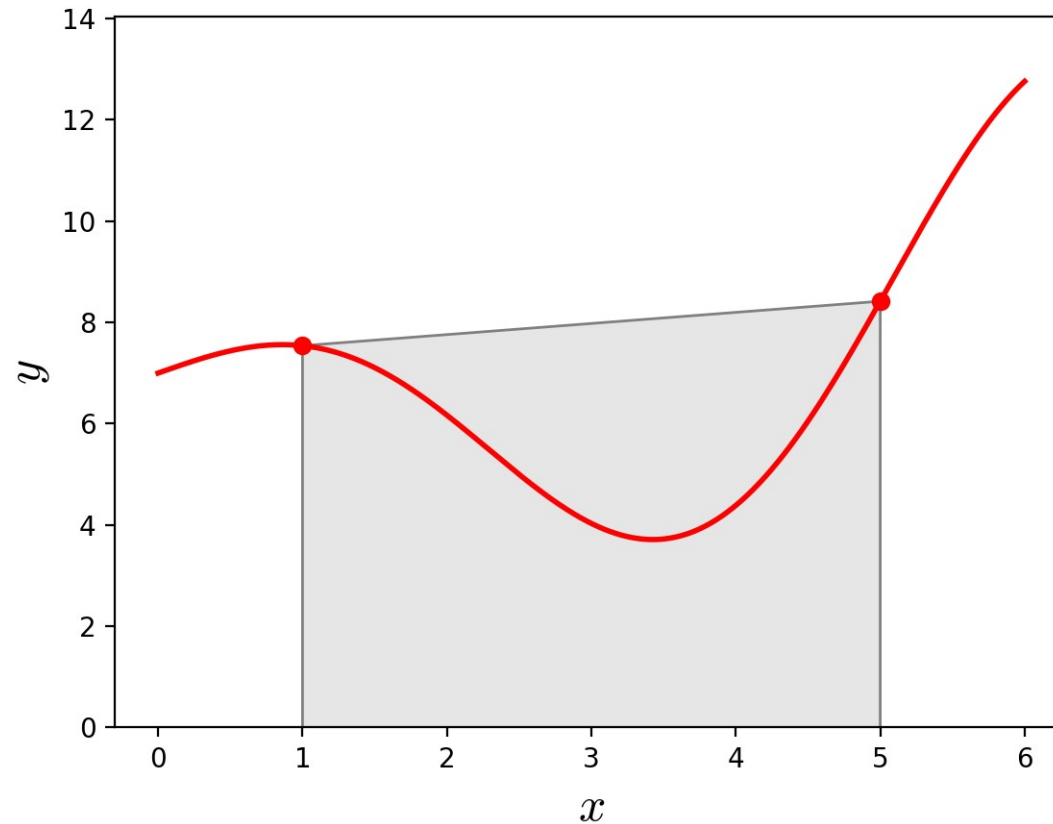


$$I[f] = 22.1073\dots \quad Q_0[f] = 16.1201\dots \quad E[f] = 5.9872\dots$$

Bsp.: (9)

Trapezregel

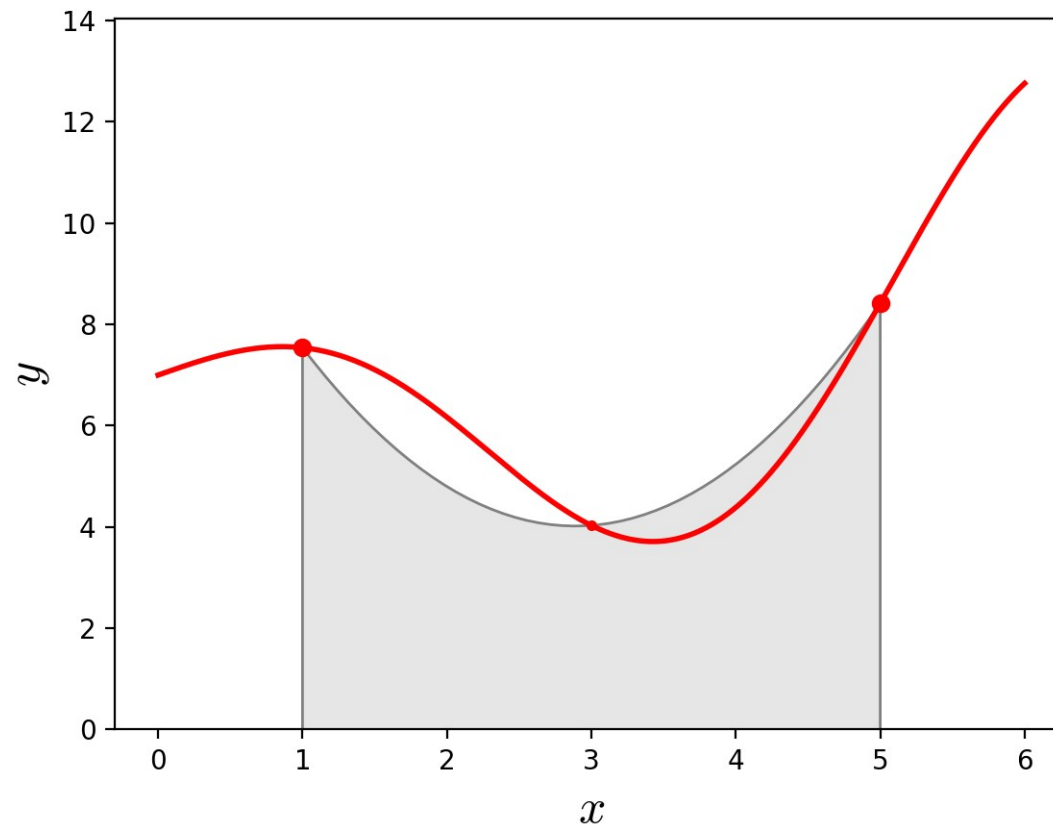
$$f(x) = x \cos(x) + 7$$



$$I[f] = 22.1073\dots \quad Q_1[f] = 31.9172\dots \quad E[f] = 9.8100\dots$$

Simpson-Regel

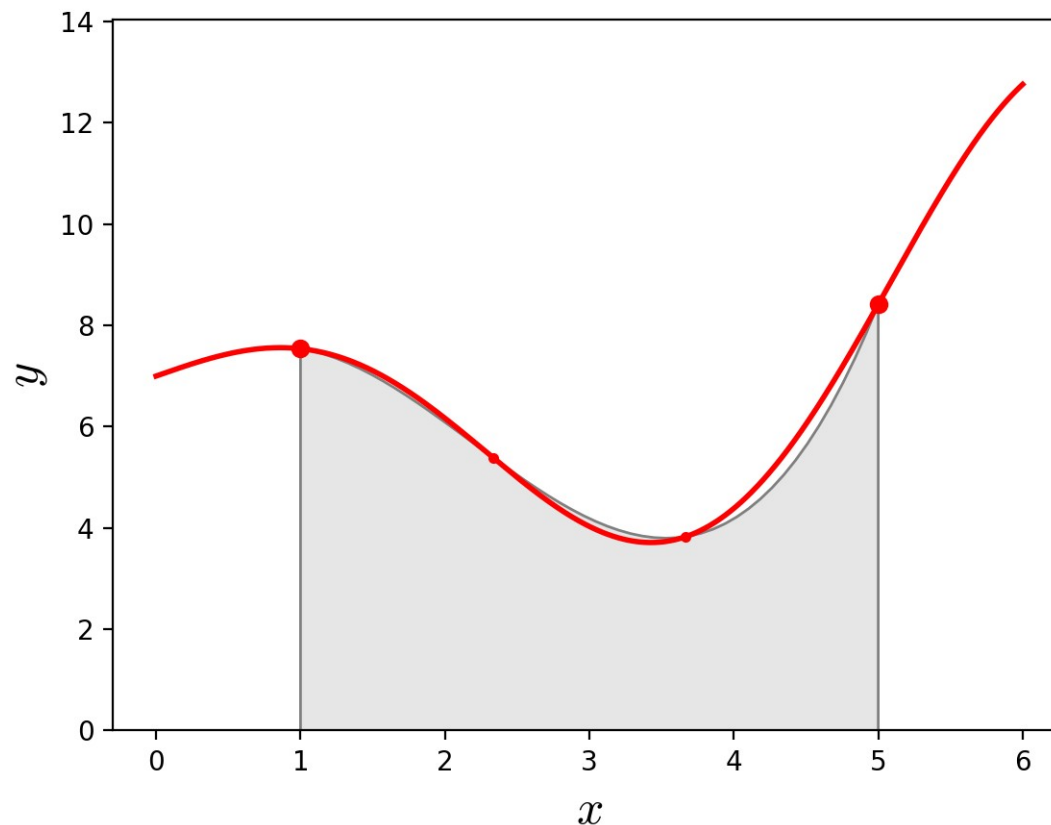
$$f(x) = x \cos(x) + 7$$



$$I[f] = 22.1073\dots \quad Q_2[f] = 21.3858\dots \quad E[f] = 0.7215\dots$$

3/8-Regel

$$f(x) = x \cos(x) + 7$$

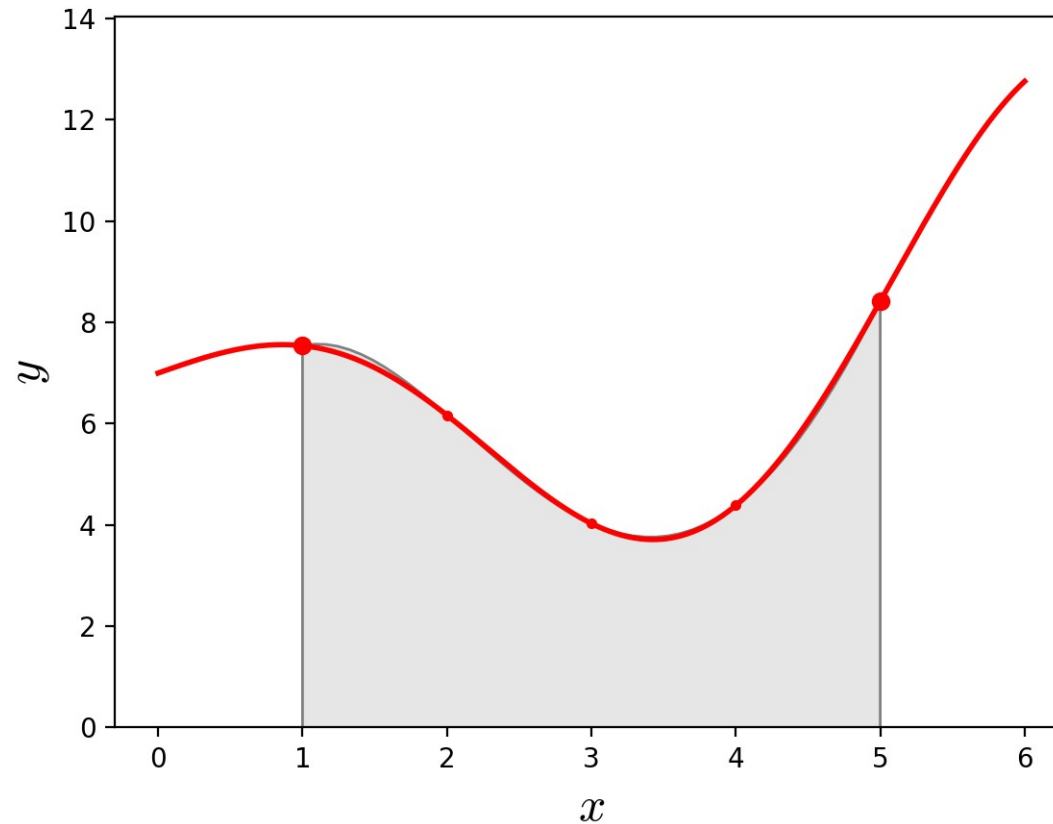


$$I[f] = 22.1073\dots \quad Q_3[f] = 21.8026\dots \quad E[f] = 0.3047\dots$$

Bsp.: (9)

Milne-Regel

$$f(x) = x \cos(x) + 7$$

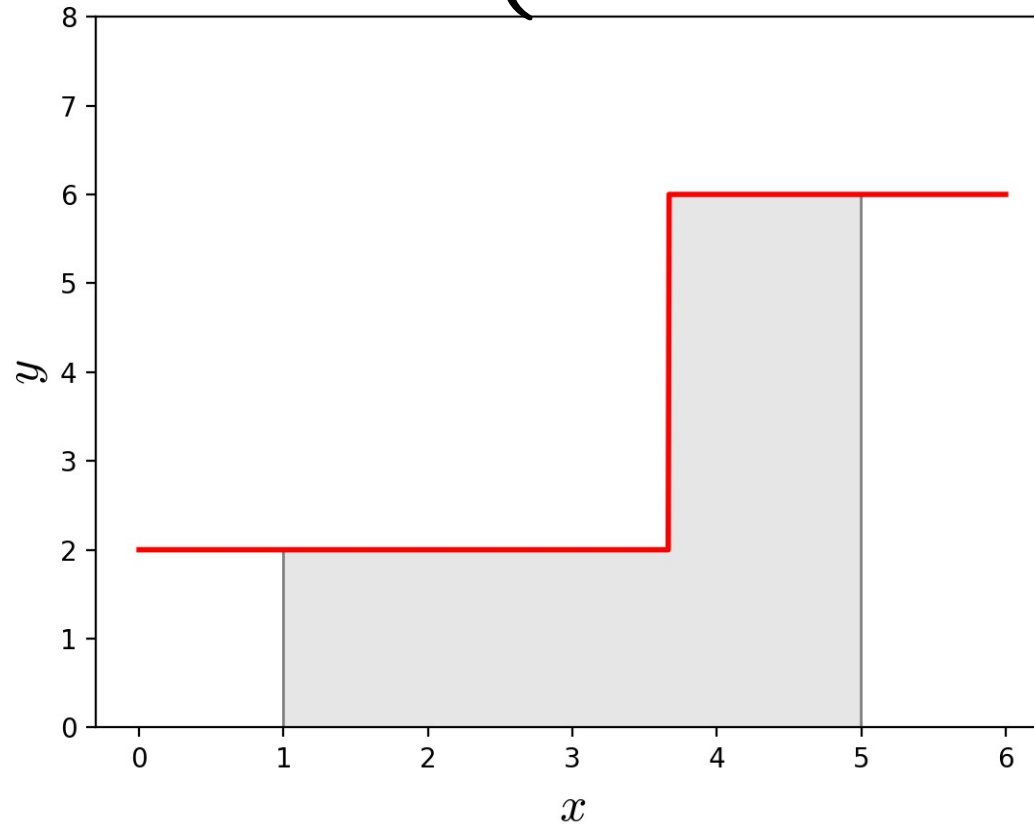


$$I[f] = 22.1073\dots \quad Q_4[f] = 22.1231\dots \quad E[f] = 0.0159\dots$$

Bsp.: (9)

Quadrature

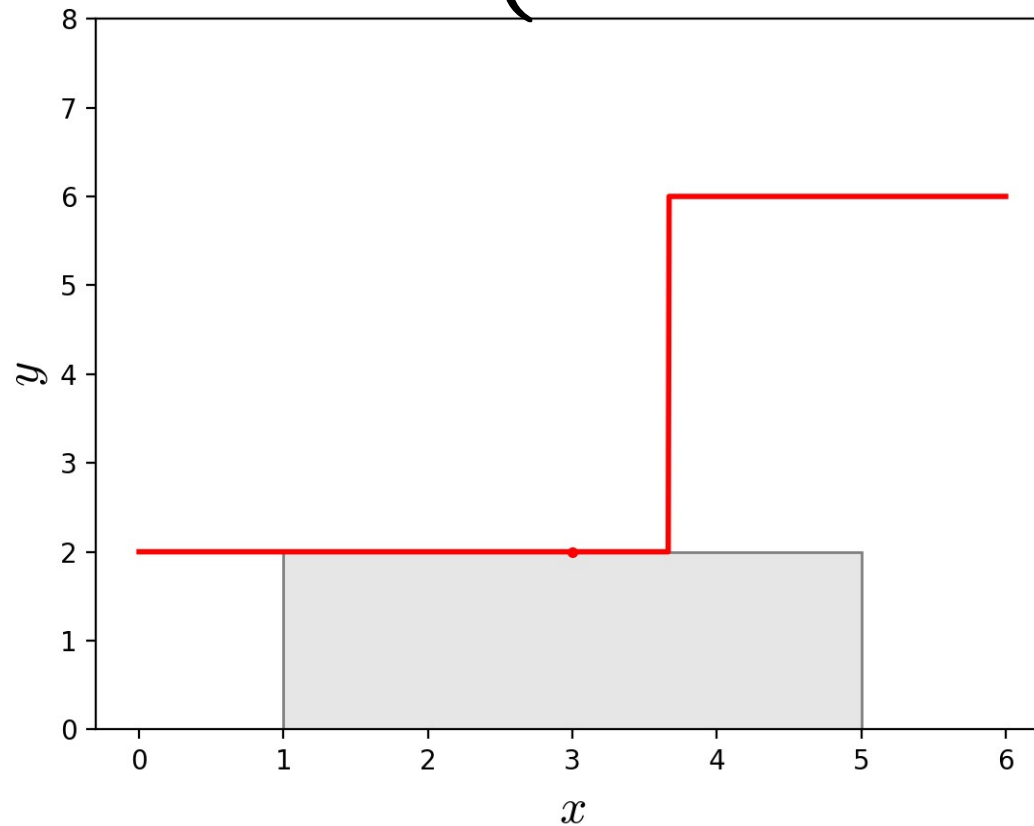
$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



$$I[f] = \int_1^5 f(x) dx = 15.4336\dots$$

Mittelpunktsregel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

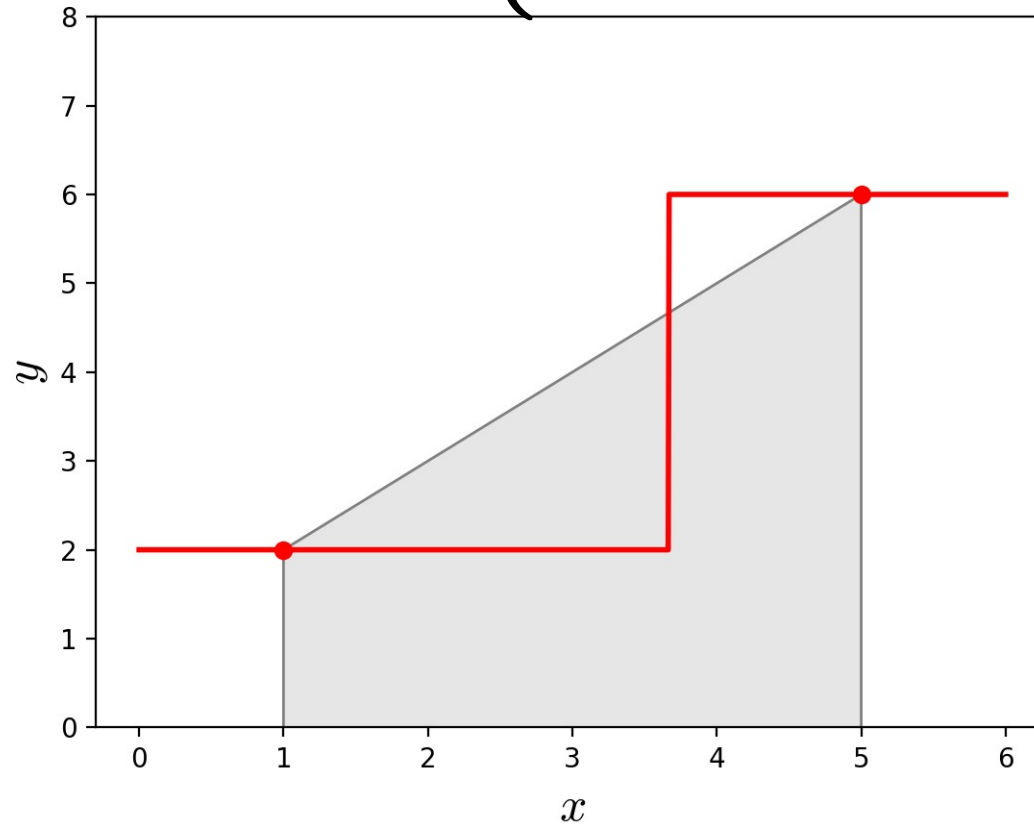


$$I[f] = 15.4336... \quad Q_0[f] = 8 \quad E[f] = 7.4336...$$

Bsp.: (9)

Trapezregel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

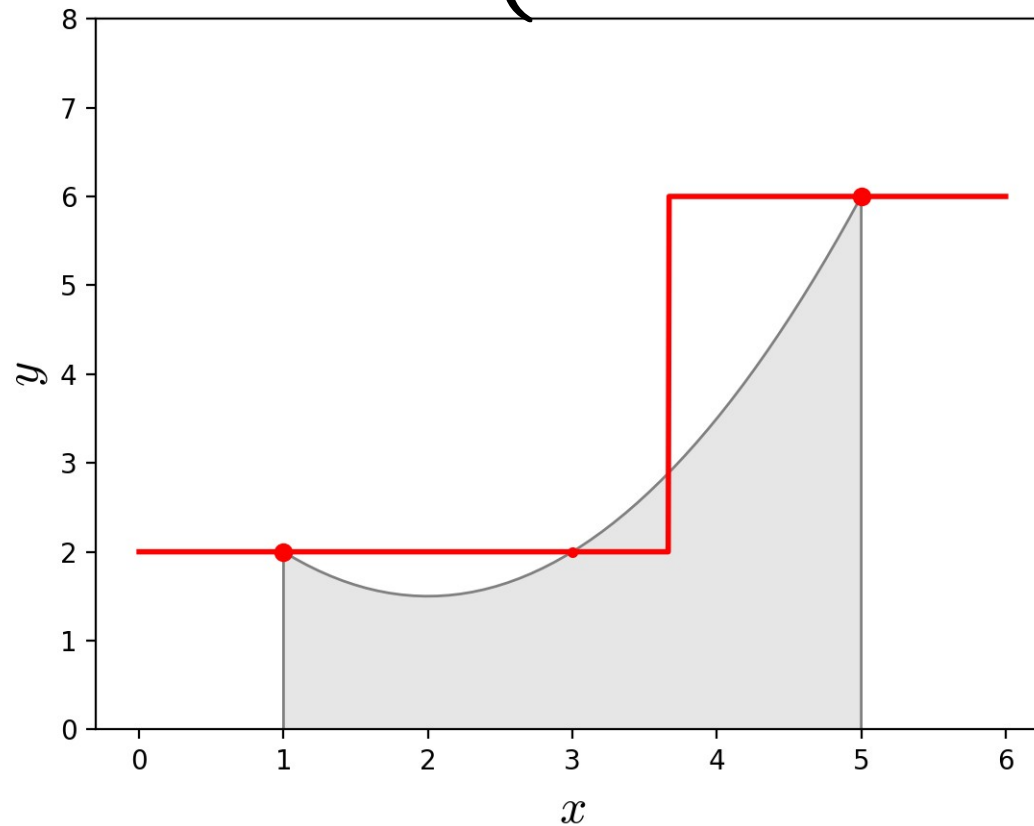


$$I[f] = 15.4336... \quad Q_1[f] = 16 \quad E[f] = 0.5664...$$

Bsp.: (9)

Simpson-Regel

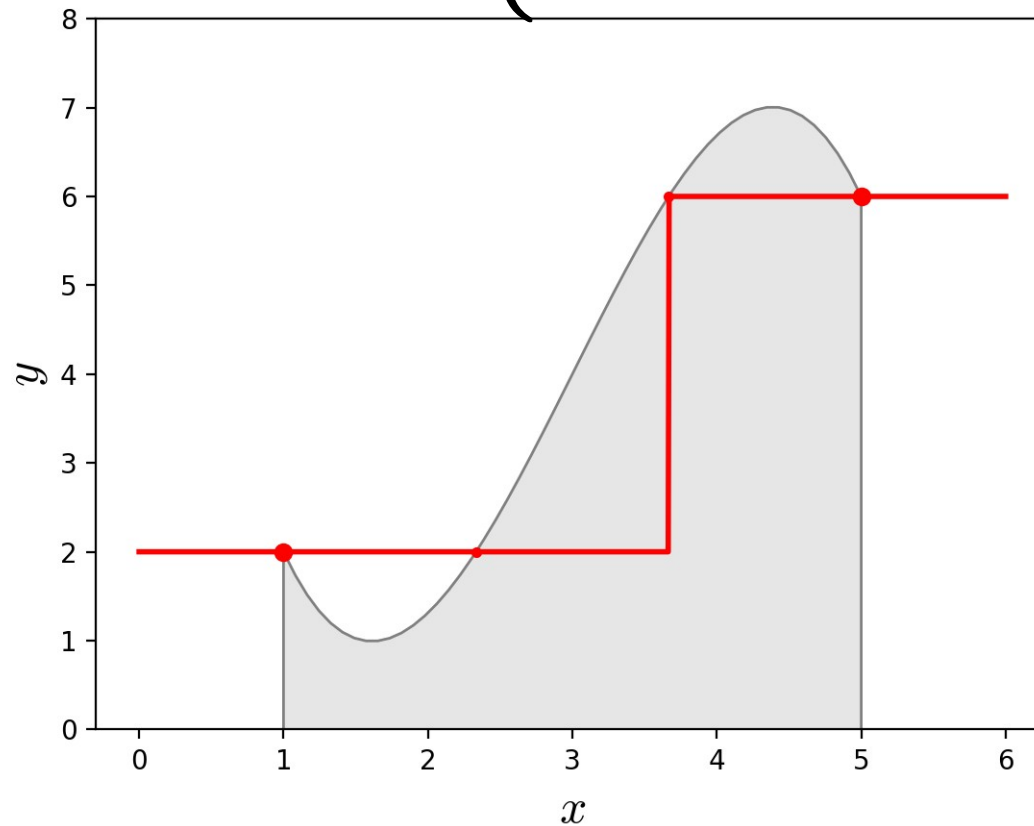
$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



$$I[f] = 15.4336\dots \quad Q_2[f] = 10.6666\dots \quad E[f] = 4.7670\dots$$

3/8-Regel

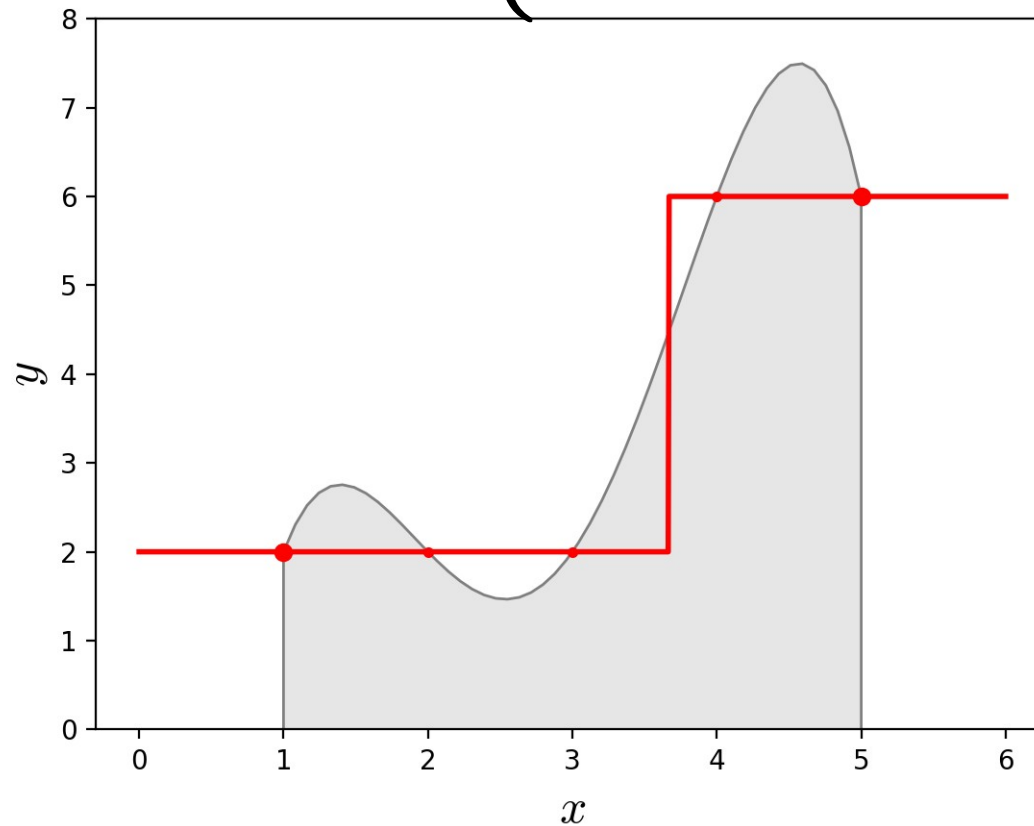
$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$



$$I[f] = 15.4336... \quad Q_3[f] = 16 \quad E[f] = 0.5664...$$

Milne-Regel

$$f(x) = \begin{cases} 2 & \text{if } x < \pi \\ 6 & \text{if } x \geq \pi \end{cases}$$

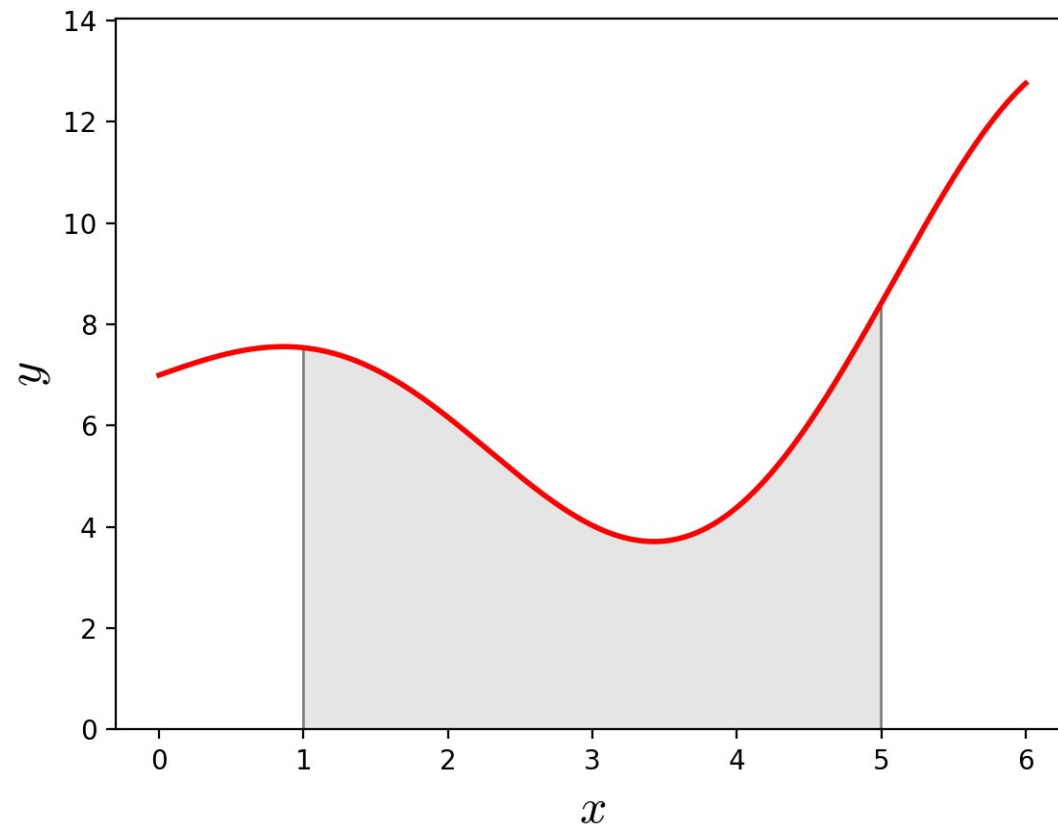


$$I[f] = 15.4336\dots \quad Q_4[f] = 14.9333\dots \quad E[f] = 0.5003\dots$$

Bsp.: (13)

Summierte Quadratur-Regeln

$$f(x) = x \cos(x) + 7$$

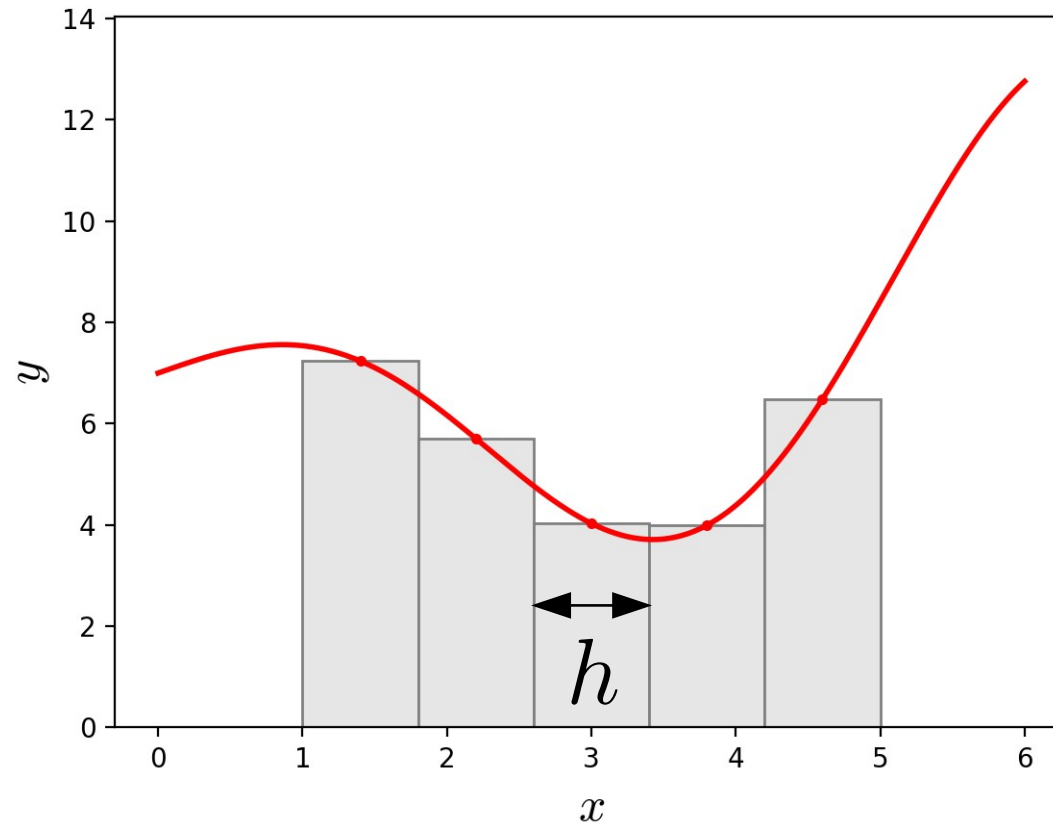


$$I[f] = \int_1^5 f(x) dx = 22.1073\dots$$

Bsp.: (13)

Summierte Mittelpunktsregel

$$f(x) = x \cos(x) + 7$$

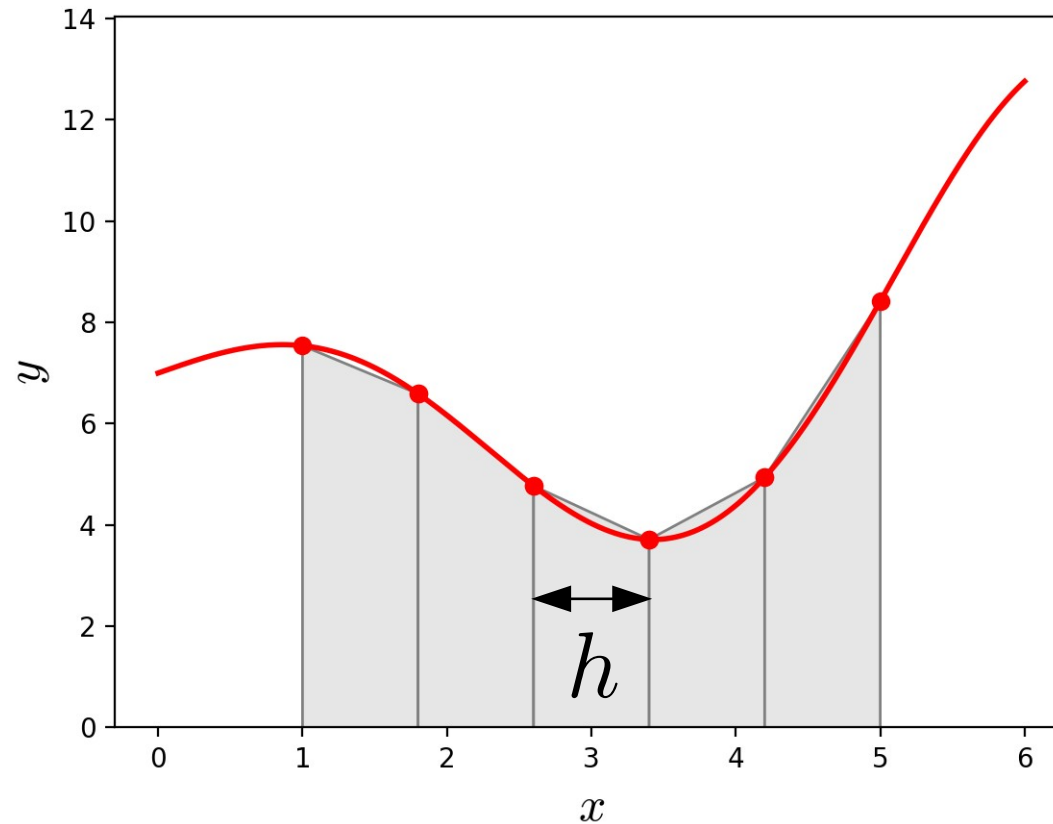


$$N = 5$$

Bsp.: (13)

Summierte Trapezregel

$$f(x) = x \cos(x) + 7$$

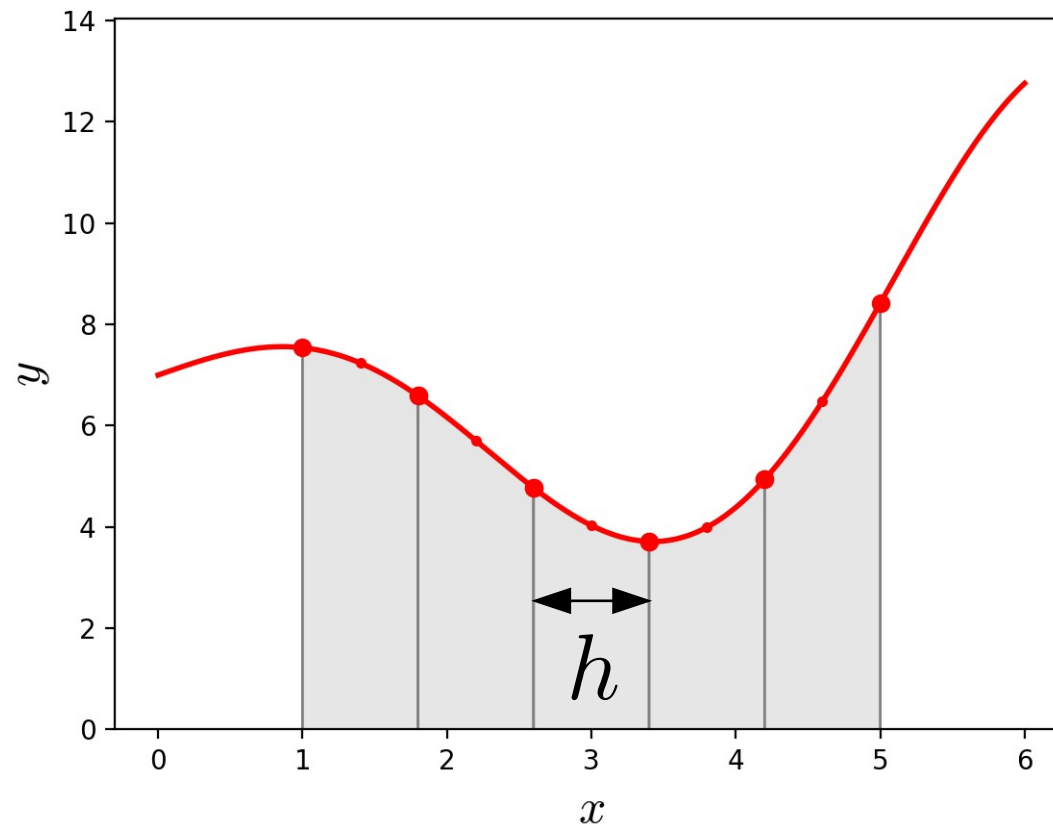


$$N = 5$$

Bsp.: (13)

Summierte Simpson-Regel

$$f(x) = x \cos(x) + 7$$

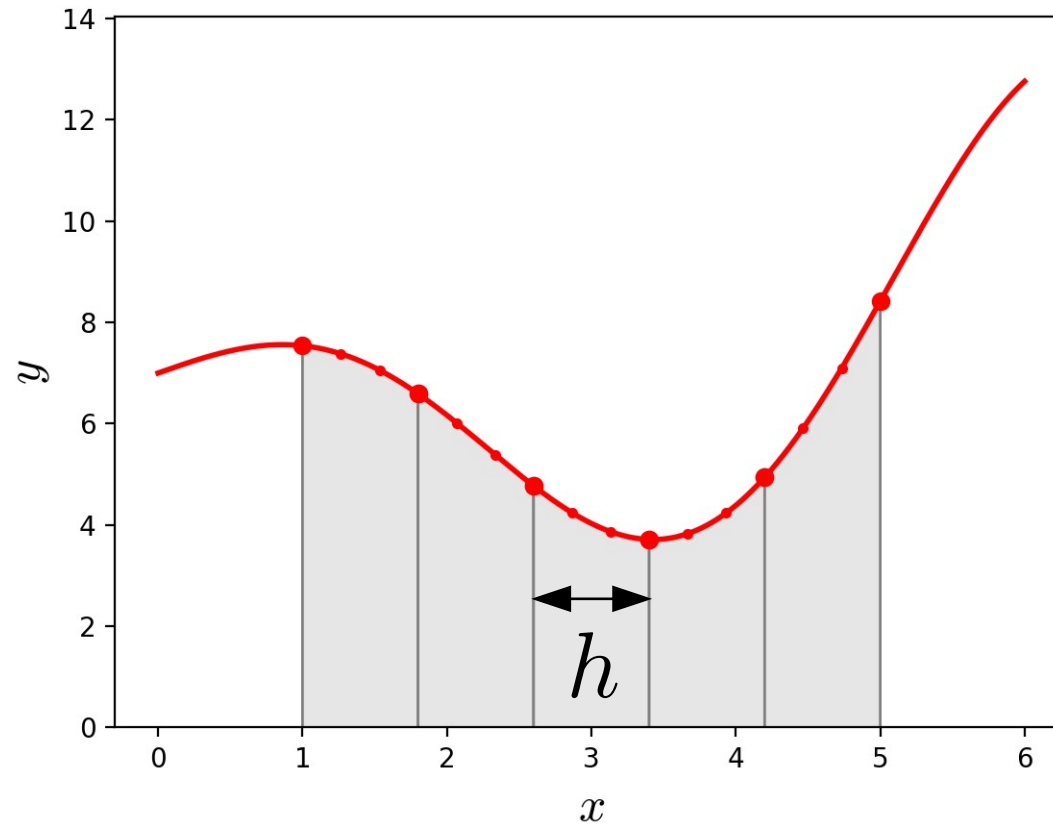


$$N = 5$$

Bsp.: (13)

Summierte 3/8-Regel

$$f(x) = x \cos(x) + 7$$

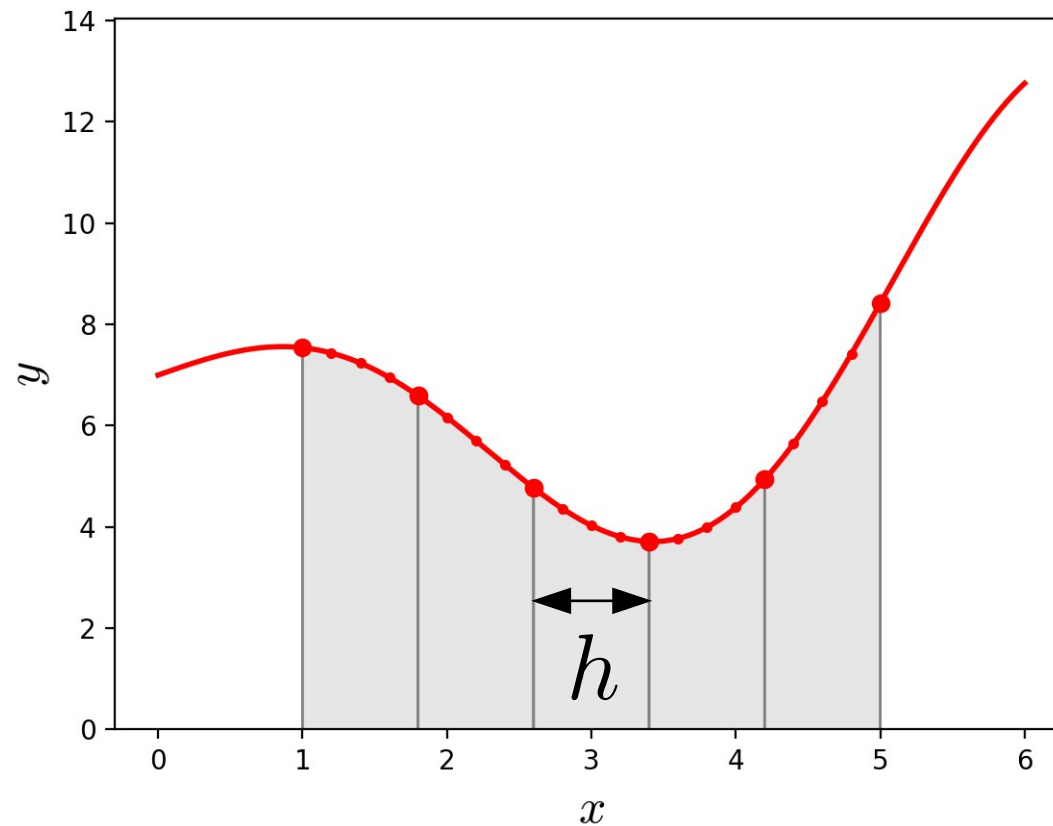


$$N = 5$$

Bsp.: (13)

Summierte Milne-Regel

$$f(x) = x \cos(x) + 7$$



$$N = 5$$

Summierte Quadratur-Regeln

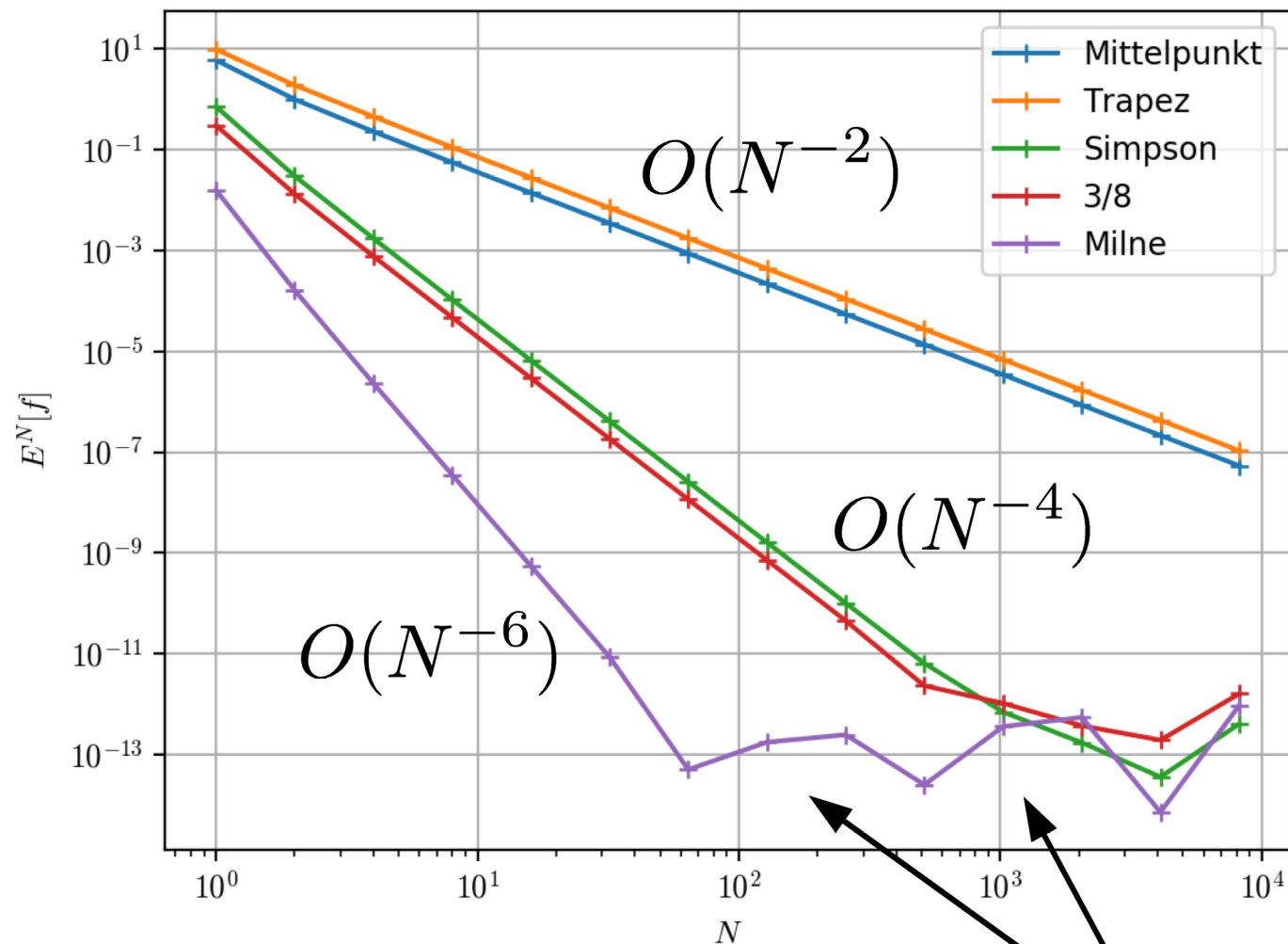
$$f(x) = x \cos(x) + 7$$

	Mittelpunkts	Trapez	Simpson	3/8	Milne	
N	h	$E_0^N[f]$	$E_1^N[f]$	$E_2^N[f]$	$E_3^N[f]$	$E_4^N[f]$
1	1.00E+00	5.99E+00	9.81E+00	7.21E-01	3.05E-01	1.59E-02
2	5.00E-01	1.00E+00	1.91E+00	3.02E-02	1.33E-02	1.62E-04
4	2.50E-01	2.30E-01	4.55E-01	1.74E-03	7.69E-04	2.32E-06
8	1.25E-01	5.64E-02	1.12E-01	1.06E-04	4.72E-05	3.54E-08
16	6.25E-02	1.40E-02	2.80E-02	6.61E-06	2.94E-06	5.50E-10
32	3.12E-02	3.50E-03	7.01E-03	4.13E-07	1.83E-07	8.62E-12
64	1.56E-02	8.76E-04	1.75E-03	2.58E-08	1.15E-08	4.97E-14
128	7.81E-03	2.19E-04	4.38E-04	1.61E-09	7.17E-10	1.74E-13

$$h = \frac{b - a}{N} \quad \dots$$

Summierte Quadratur-Regeln

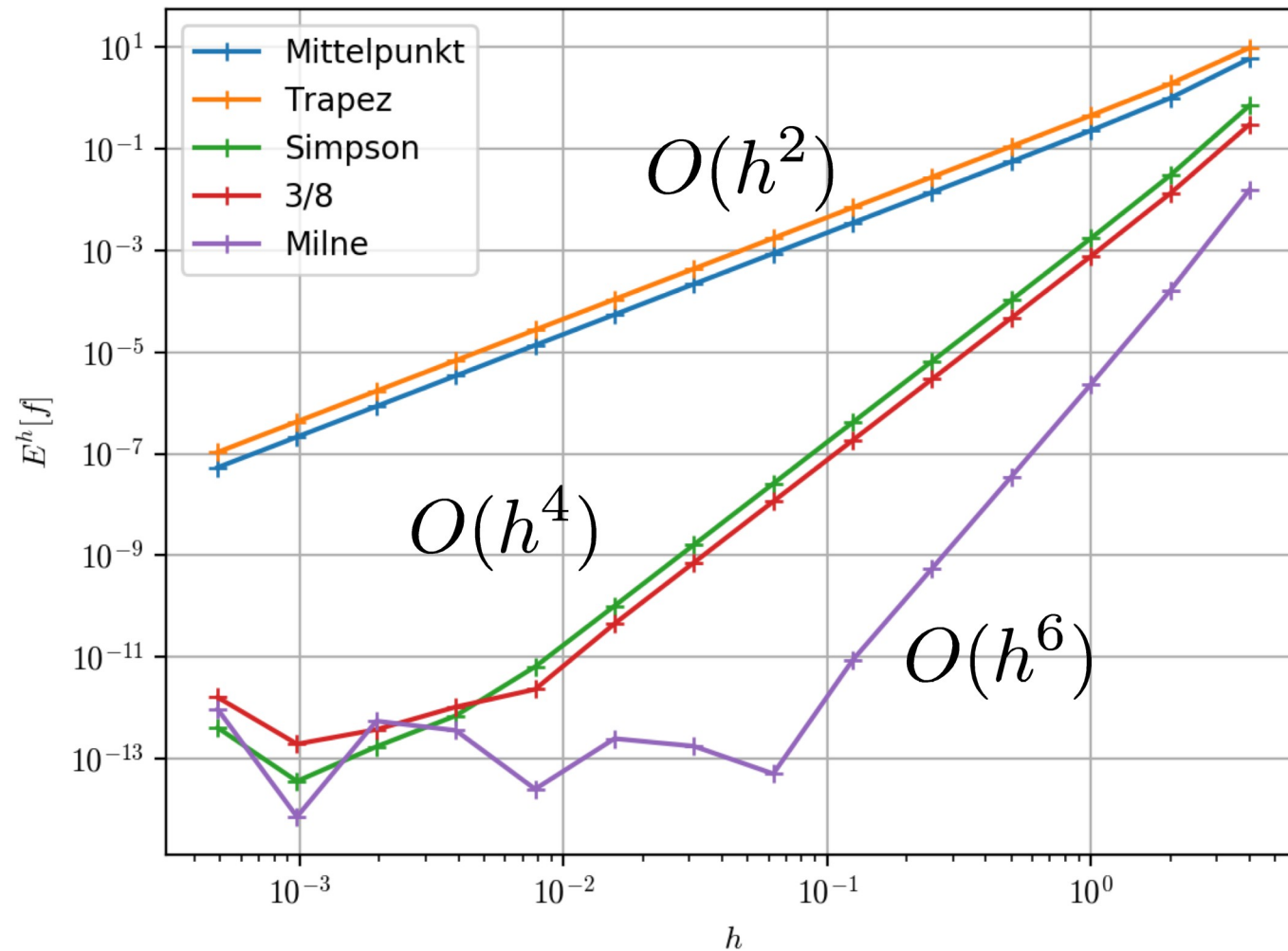
$$f(x) = x \cos(x) + 7$$



Maschinengenauigkeit!

Summierte Quadratur-Regeln

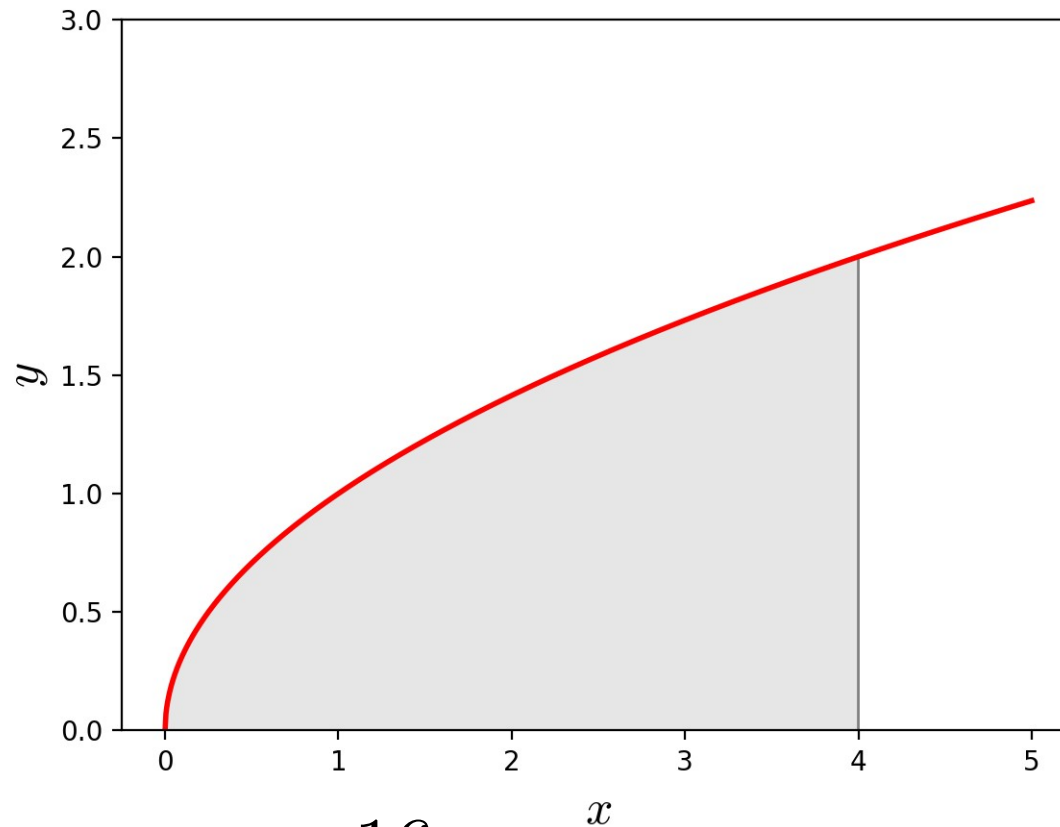
$$f(x) = x \cos(x) + 7$$



Bsp.: (13)

Summierte Quadratur-Regeln

$$f(x) = \sqrt{x}$$

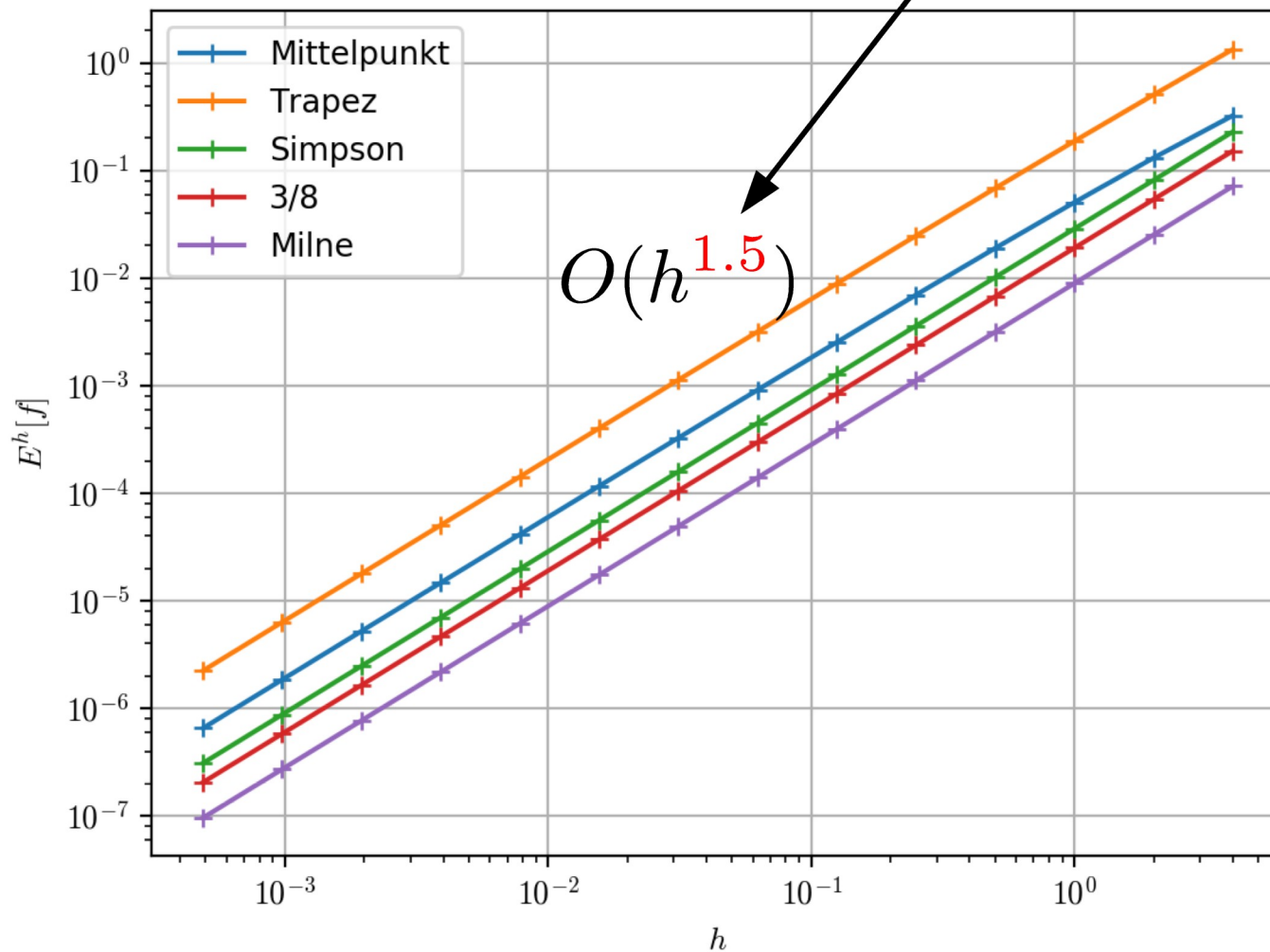


$$I[f] = \int_0^4 f(x) dx = \frac{16}{3}$$

Summierte Quadratur-Regeln

$$f(x) = \sqrt{x}$$

$$? f'(x) = \frac{1}{2\sqrt{x}} \xrightarrow{x \rightarrow 0} \infty$$



...Glattheit!

Fehlerschätzer

Bsp.: (15) $I[e^x] = \int_0^1 e^x dx = e - 1 = 1.71828\dots$

$$TR: Q_1[e^x] = \frac{1}{2} (e^0 + e^1) = 1.85914\dots$$

$$SR: Q_2[e^x] = \frac{1}{6} (e^0 + 4 \cdot e^{1/2} + e^1) = 1.71886\dots$$

$$STR: Q_1^2[e^x] = \frac{1/2}{2} (e^0 + 2 \cdot e^{1/2} + e^1) = 1.75393\dots$$

Exakter Fehler : $|Q_1[e^x] - I[e^x]| = 0.14085\dots$

Schätzung 1 : $|Q_1[e^x] - Q_2[e^x]| = 0.14027\dots$

Schätzung 2 : $|Q_1[e^x] - Q_1^2[e^x]| = 0.10520\dots$

Adaptive Quadratur Pseudo-Code

Adaptive Quadratur (~ Pseudo-MATLAB-Code)

function $Q = \text{adapt_quad}(f, a, b, \text{tol})$

if $E < \text{tol}$

$Q = \text{quad}(f, a, b)$

else

$Q_1 = \text{adapt_quad}(f, a, \frac{a+b}{2}, \frac{\text{tol}}{2})$

$Q_2 = \text{adapt_quad}(f, \frac{a+b}{2}, b, \frac{\text{tol}}{2})$

$Q = Q_1 + Q_2$

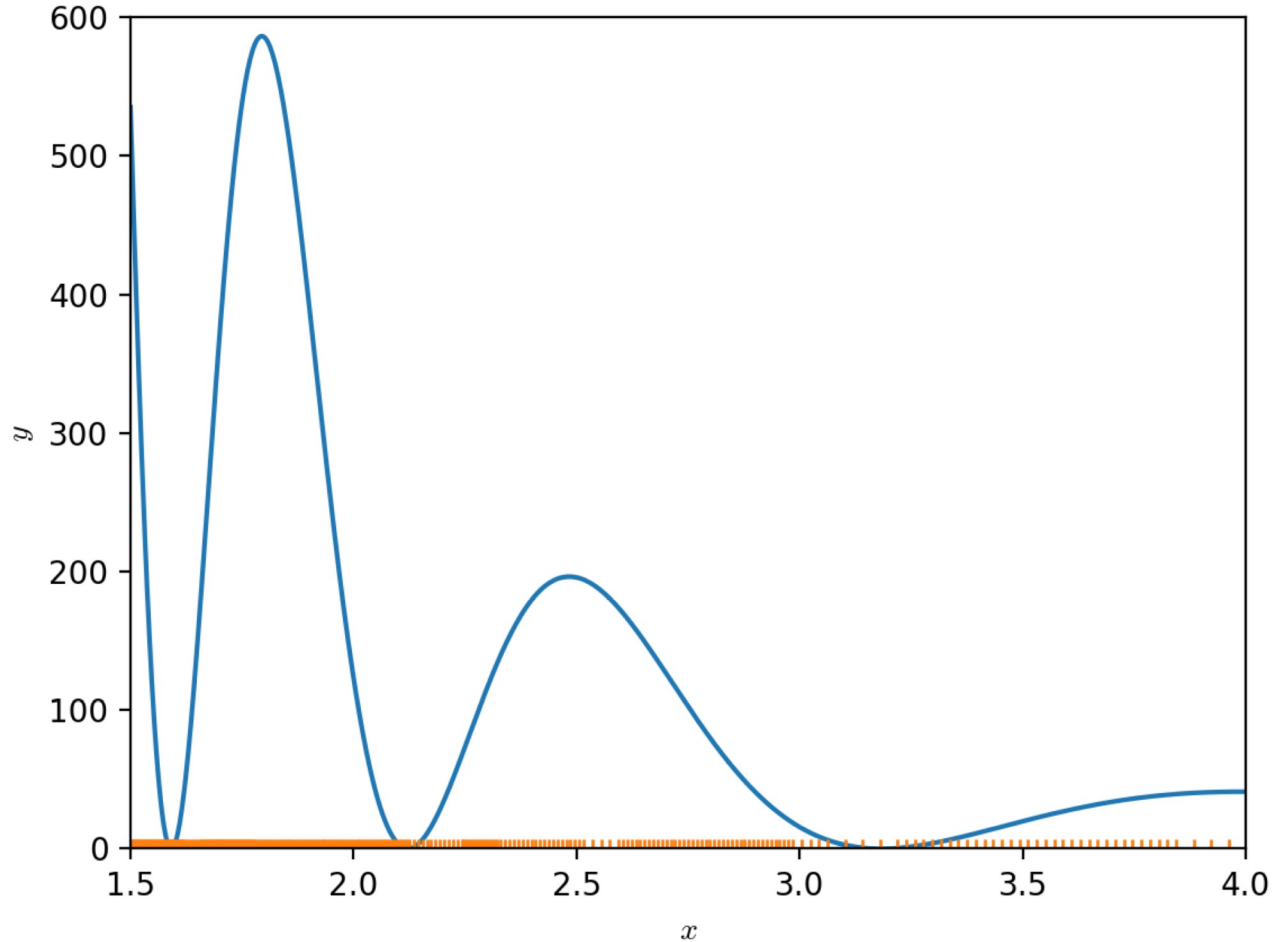
end

↑
halbieren
Fehler-Toleranz, da nun
halbiertes Int.-Intervall!

Bsp.: (17)

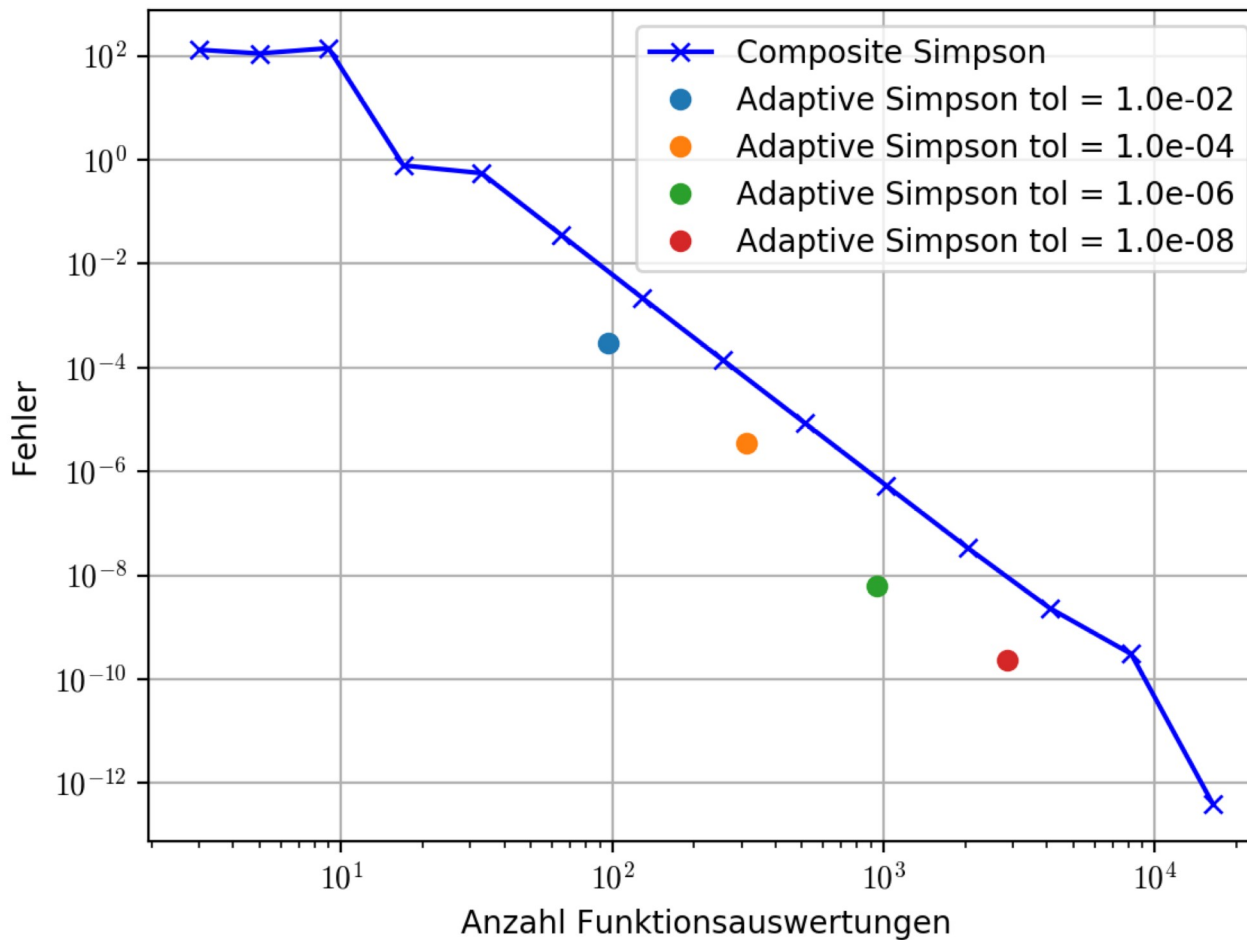
Adaptive Quadratur

$$I[f] = \int_{3/2}^4 \left(\frac{200}{2x^3 - x^2} \right) (5 \sin(20/x))^2 dx$$



Adaptive Quadratur

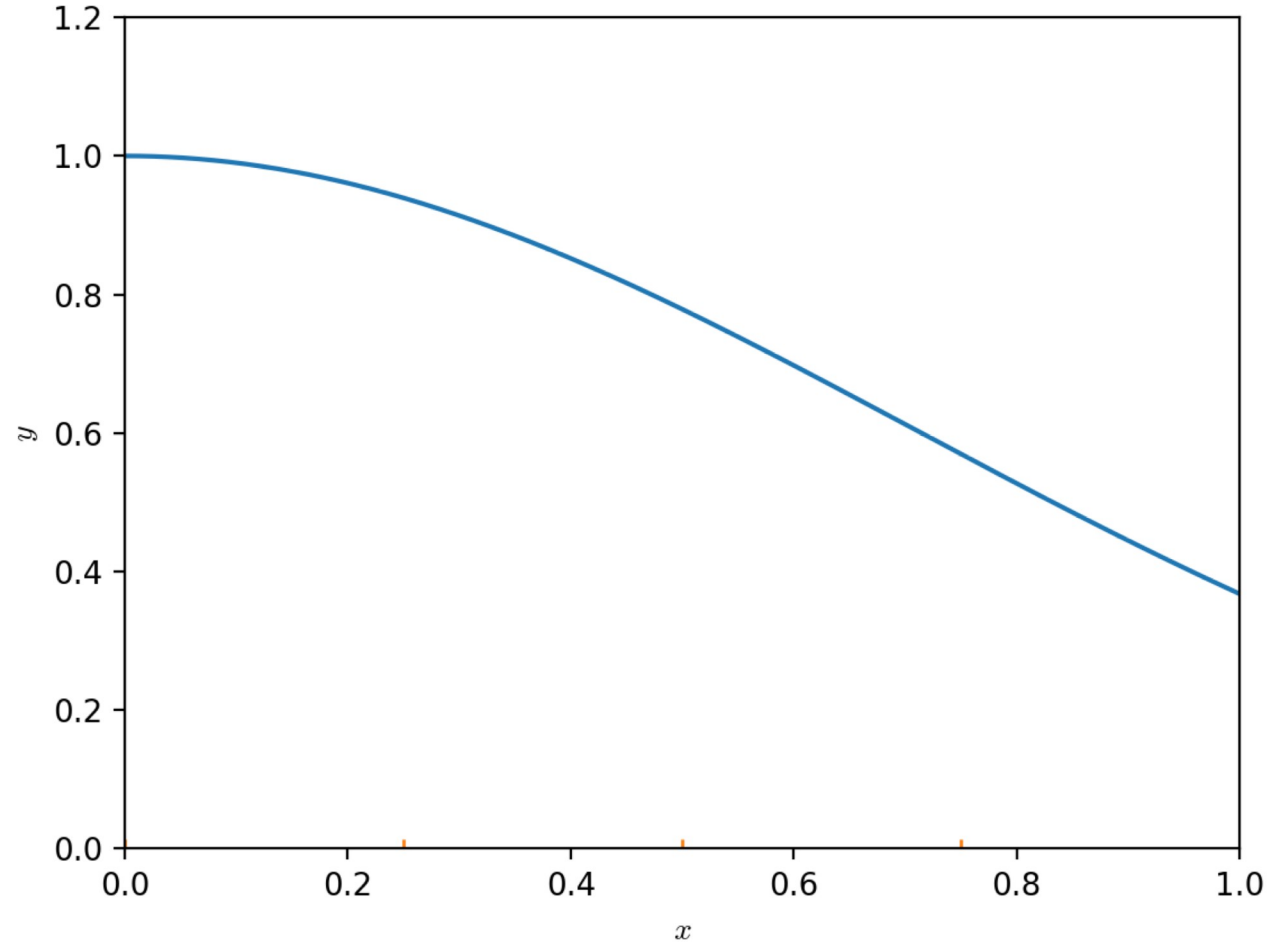
$$I[f] = \int_{3/2}^4 \left(\frac{200}{2x^3 - x^2} \right) (5 \sin(20/x))^2 dx$$



Bsp.: (17)

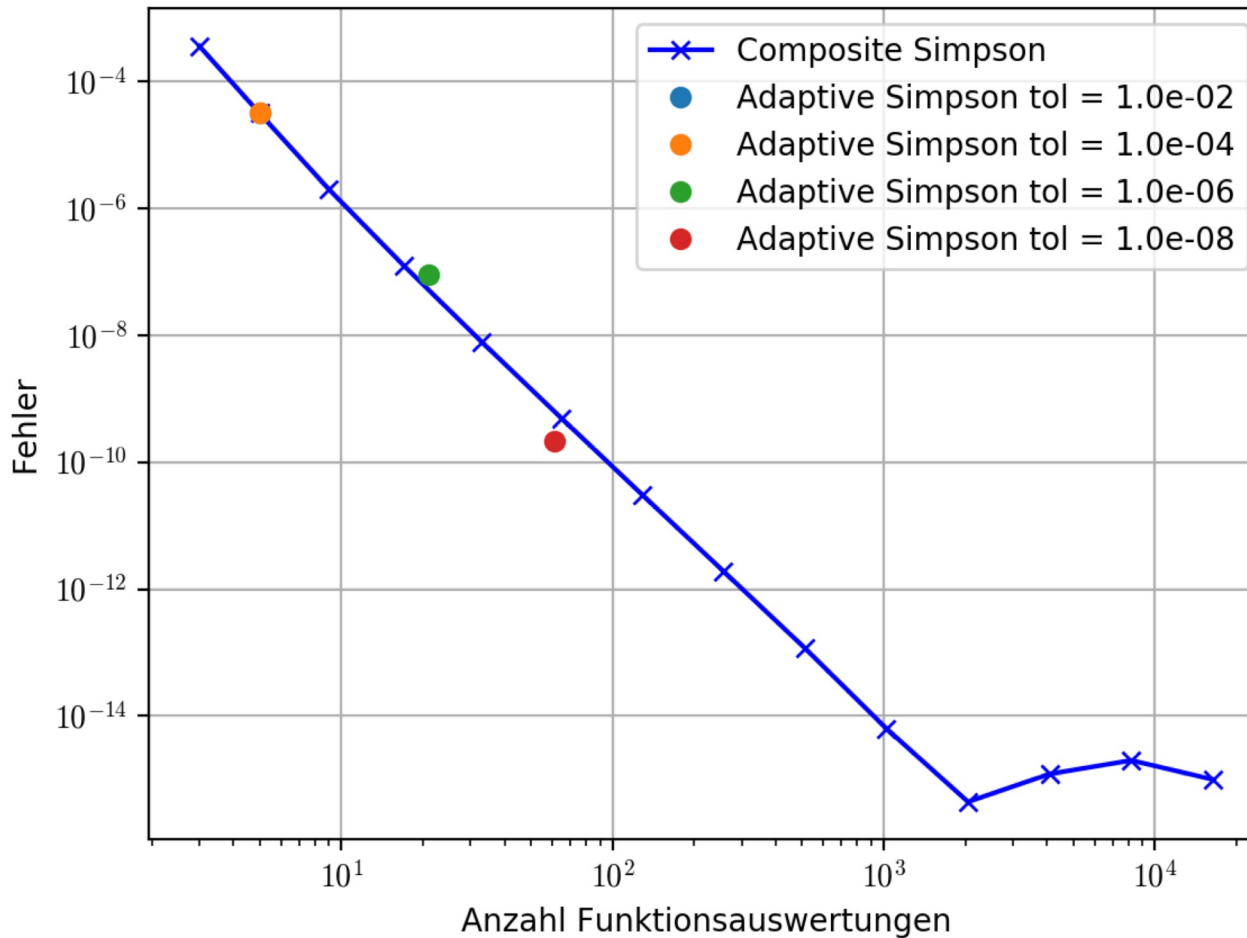
Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x^2} dx$$



Adaptive Quadratur

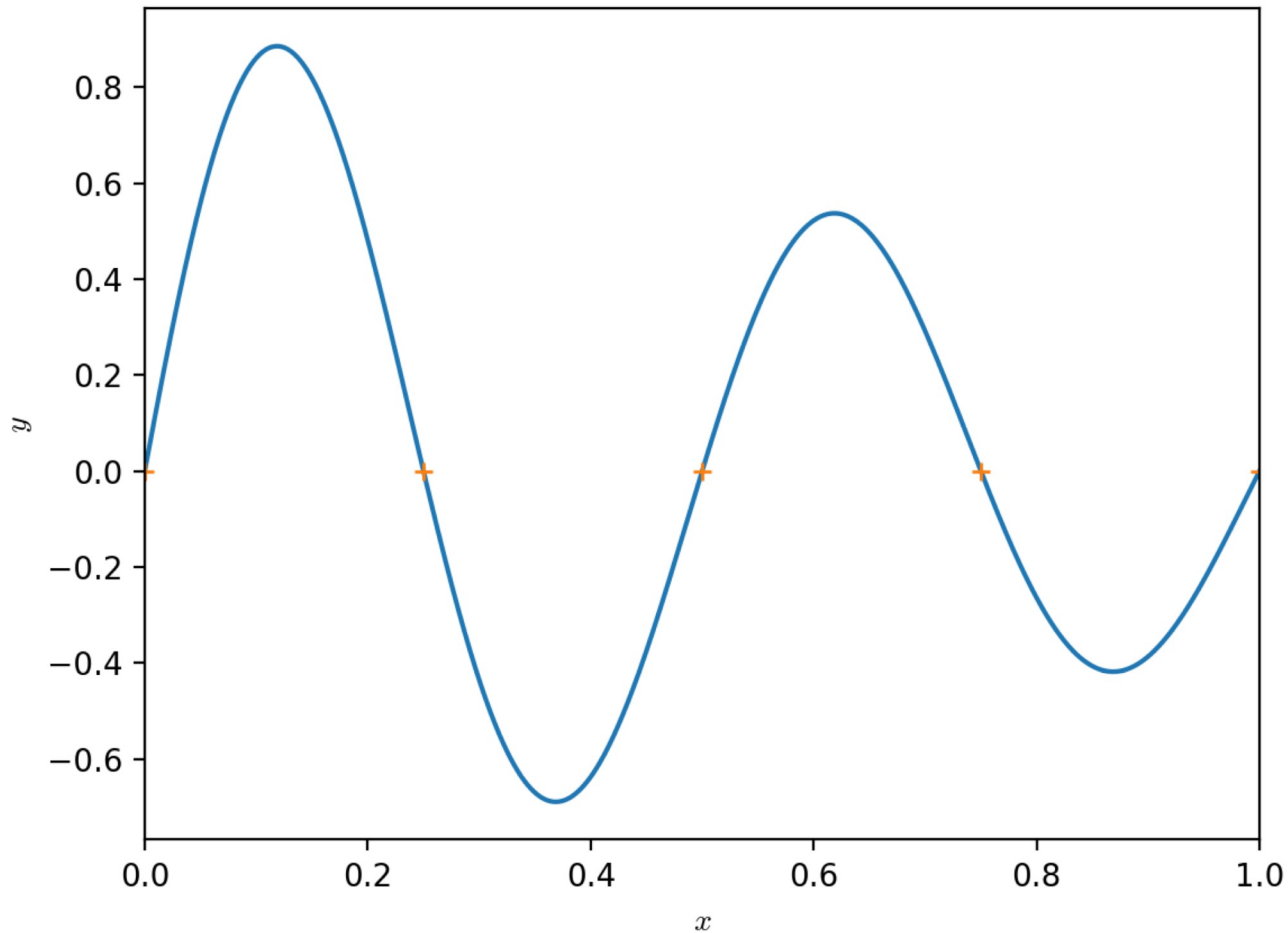
$$I[f] = \int_0^1 e^{-x^2} dx$$



Bsp.: (17)

Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x} \sin(4\pi x) dx$$



Adaptive Quadratur

$$I[f] = \int_0^1 e^{-x} \sin(4\pi x) dx$$

