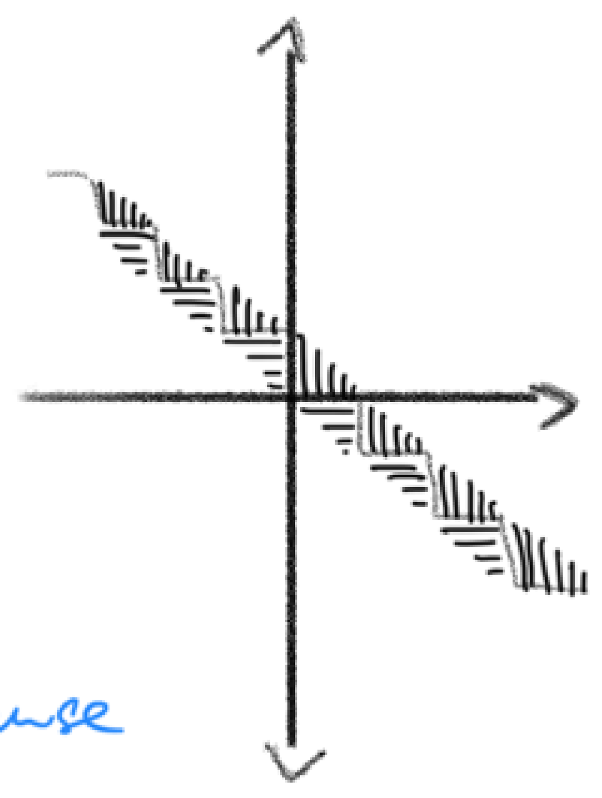
Consider the space displayed in the figure imported from Hatcher's notes.

We parametrize the space X as follows:

$$\Omega_1 = \{r\} \times [0, 1-r]$$

$$\Omega_2 = [r, 1] \times \{r-1\}$$

$$X = \bigcup_{z \in \mathbb{Z}} (z,z) + (\Omega_1 \cup \Omega_2)$$



Note that X is <u>not deformation retractable</u> to any point $x \in X$ because $\forall x \in X : \exists z \in \mathbb{Z} : \; x \in (z,z) + \Omega_1$ or $x \in (z,z) + \Omega_2$

and since $(z,z) + \Omega_i$ is the translated and rotated set of the vertical segments as defined in 8.9.2 ($(z,z) + \Omega_i = T(\{\text{vertical segments}\})$ for some cont. T) assuming the existence of a deformation retraction $H : X \times I \longrightarrow X$ onto x

$$T^{-1} \circ H(T(\cdot), \cdot) : \tilde{X} \times I \longrightarrow \tilde{X},$$

where $\tilde{X}$ is the union of the vertical segments and the horizontal segment as defined in 8.9.2, would define a deformation retraction onto $T^{-1}(x)$ $\lightning$ (8.9.2)

Consider the following <u>contraction</u>: $\forall z \in \mathbb{Z} : \forall r \in \mathbb{Q} \cap [0,1]$

$\forall x \in (z,z) + [r,1] \times \{r-1\}$:

$$\forall t \in [0,1]: \; h_x(t) = (z,z) + \begin{cases} (x_1 + t, x_2) & \text{if } t \leq 1 - x_1 \\ (1, x_2 - (t-(1-x_1))) & \text{if } 1-x_1 \leq t \text{ and } t-(1-x_1) \leq r \\ (1+t-(1-x_1)-r, 0) & \text{else} \end{cases}$$

$x = (z,z) + (x_1, x_2)$

is cont.

$\forall x \in (z,z) + \{r\} \times [0, 1-r]$:

$$\forall t \in [0,1]: \; h_x(t) = (z,z) + \begin{cases} (x_1, x_2 - t) & \text{if } t \leq x_2 \\ (x_1 + (t - x_2), 0) & \text{if } x_2 \leq t \text{ and } t - x_2 \leq 1-r \\ (1, -(t-x_2-(1-r))) & \text{else} \end{cases}$$

is cont.

Consider $H : X \times I \longrightarrow X$

$$H(x,t) = h_x(t).$$

Note that $\forall t \in I : H(\cdot, t)$ is Lipschitz:

$\Rightarrow H$ is cont.

and in particular H is a homotopy s.t.

$H(x,0) = x$ and $\operatorname{im}(H(\cdot,1)) = \bigcup_{z \in \mathbb{Z}} (z,z) + ([0,1] \times \{0\} \cup \{0\} \times [0,1])$

Since

$\bigcup_{z \in \mathbb{Z}} (z,z) + ([0,1] \times \{0\} \cup \{0\} \times [0,1])$ is homeomorphic to $\mathbb{R}$ and $\mathbb{R}$ is contractible

X is contractible. //