Probability and Statistics

Exercise sheet 10

Please ask questions in the exercise classes and/or post your questions (anonymously if you want) in this file: https://docs.google.com/document/d/1dJ48c74AXtwIUzSew_wJ_J0z_VjT5EZ8GcUr1TCge9M/edit?usp=sharing

Exercise 10.1 Seien $X_2, X_3, \ldots$ unabhängige Zufallsvariablen mit $\mathbb{P}[X_n = n] = \mathbb{P}[X_n = -n] = \frac{1}{2n \log n}$ und $\mathbb{P}[X_n = 0] = 1 - \frac{1}{n \log n}$, $n = 2, 3, \ldots$. Zeige, dass die Folge $(X_n)_{n \geq 2}$ das schwache, aber nicht das starke Gesetz der grossen Zahlen erfüllt.

Exercise 10.2 We want to estimate the number of a certain type of fish in a lake. For this, we mark 5 fish and we let them mix with the others. When they are well mixed, we fish 11, and we observe that there are 3 marked and 8 non-marked. To estimate the total number of fish N, compute the probabilities of this outcome as a function of N and then take as estimated value N^* for N that N for which this probability becomes maximized. In other words, choose N to give the observed data maximal probability. Which N^* do you find?

Exercise 10.3 Let $X_i, \ldots X_n$ be a sequence of i.i.d. Bernoulli-distributed random variables with unknown parameter p. We consider two different estimators. The first estimator is X_1. The second estimator is $\overline{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$.

(a) Compute the expected value and the variance as a function of n for both estimators. Comment on the result.

(b) Are these estimators unbiased?

(c) Are these estimators consistent? (More precisely: Are the sequences of estimators $(X_1)_{n \in \mathbb{N}}$ and $(\overline{X}_n)_{n \in \mathbb{N}}$ consistent?)

Exercise 10.4 Let $X_i, \ldots X_n$ be a sequence of i.i.d. exponential random variables with unknown parameter α. The so-called moment estimator for α is given by $\hat{\alpha}_n := \frac{1}{\overline{X}_n} = \frac{n}{\sum_{i=1}^n X_i}$.

(a) Is this estimator unbiased?

(b) Is this estimator consistent? (More precisely: Is the sequence of estimators $(\hat{\alpha}_n)_{n \in \mathbb{N}}$ consistent?)

If you have feedback regarding the exercise sheets, please send a mail to Jakob Heiss.