# Probability and Statistics

## Exercise sheet 2

Please ask questions in the exercise classes and/or post your questions (anonymously if you want) in this file: https://docs.google.com/document/d/1xObbQFXf6GsiarNAJ46pkpOhbzPM5PPXDqkZxsoFA10/edit?usp=sharing

**Exercise 2.1** Let $X : \Omega \to \mathbb{R}_+$ be a nonnegative discrete random variable taking its values in the set $\{x_1, x_2, \ldots\}$ (possibly countably infinite), where we assume that the values are ordered as $x_1 < x_2 < \cdots$. Show that

$$\mathbb{E}[X] = \sum_{j=0}^{\infty} (x_{j+1} - x_j)\mathbb{P}[X > x_j] \tag{1}$$

with $x_0 := 0$.

How does this connect to equation (1.12) from the lecture notes?

**Exercise 2.2** In a building with 6 floors (plus the ground floor), an elevator starts with 4 people at the ground floor. What is the probability that these people get off at exactly 2 floors?

**Exercise 2.3** Consider Beispiel 2.1. Garderobenproblem from the lecture notes, where $n$ coats are distributed randomly to $n$ persons. We assume a Laplace model on the set $\Omega$ of all permutations of $\{1, \ldots, n\}$ as explained in the lecture notes. What is the expected number of persons who get their own coat back—i.e. $\mathbb{E}[X]$ with $X(\omega) := |\{i \in \{1, \ldots, n\} : \omega(i) = i\}|$?
*Hint:* Use the properties of expectation. Do not try to find the distribution of $X$.

**Exercise 2.4** One coin is flipped and one die is rolled.

(a) Define a suitable probability space $(\Omega, \mathcal{F}, \mathbb{P})$ space using a Laplace model.

(b) Define random variables $X : \Omega \to \mathbb{R}$ and $Y : \Omega \to \mathbb{R}$ on this probability space such that $X$ and $Y$ represent the outcome of flipping the coin and of the roll of the die, respectively.

(c) Show that $\mathbb{P}[X = x, Y = y] = \mathbb{P}[X = x]\mathbb{P}[Y = y]$ for all $x, y \in \mathbb{R}$. (This means that the random variables $X$ and $Y$ are independent; see later.)

If you have feedback regarding the exercise sheets, please send a mail to Jakob Heiss.