# Probability and Statistics

## Exercise sheet 3

Please ask questions in the exercise classes and/or post your questions (anonymously if you want) in this file: https://docs.google.com/document/d/1FuW9HQponei5ipS4j2J3lM4dfiP7MVQQ_0cMB1UUAXg/edit?usp=sharing

**Exercise 3.1** We have two dice. One is ordinary with the numbers $1, 2, 3, 4, 5, 6$ and one is special where 6 is replaced by 7 (i.e. $1, 2, 3, 4, 5, 7$). We flip a coin to decide which die is rolled. If flipping the coin results in heads, the ordinary die is rolled, otherwise the special die is rolled.

(a) Define a suitable probability space $(\Omega, \mathcal{F}, \mathbb{P})$ using a Laplace model.

(i) Define random variables $X : \Omega \to \mathbb{R}$ and $Y : \Omega \to \mathbb{R}$ such that $X$ and $Y$ represent the outcomes of flipping the coin and of the roll of the die, respectively.

(ii) What is the cardinality $|\mathcal{F}|$? Give examples of events $E_1, E_2, E_3, E_4 \in \mathcal{F}$ such that $\mathbb{P}[E_i] \neq \mathbb{P}[E_j]\,,\ \forall i \neq j$.

(b) What is the probability that rolling the die results in an even number?

(c) Show that $\mathbb{P}[X = x, Y = y] \neq \mathbb{P}[X = x]\,\mathbb{P}[Y = y]$ for some $x, y \in \mathbb{R}$. (This means that the random variables $X$ and $Y$ are not independent; see later and cf. Exercise 2.4 from Excercise Sheet 2.)

**Exercise 3.2** Let $(S_n)_{n=0,1,\dots,2N}$ be a simple random walk on $\{-1, 1\}^{2N}$. Fix $a, b \in \mathbb{Z}$

(a) Prove that $T_a := \min\{n \in \mathbb{N}_0 : S_n = a\} \wedge 2N$ is a stopping time.

(b) Prove that $\tau_b := \min\{n > T_a : S_n = b\} \wedge 2N$ is a stopping time.

(c) Show that $L = \max\{0 \leq n \leq 2N : S_n = 0\}$ is not a stopping time if $N \geq 1$.

**Exercise 3.3** Let $(S_n)_{n=0,1,\dots,N}$ be a simple random walk on $\{-1, 1\}^N$. Consider the strategy of first betting 1 and then successively tripling your bet until you win for the first time.

(a) Describe this strategy by a gambling strategy $V$.

(b) Calculate the resulting total income $(V \bullet S)_N$ and its expected value $\mathbb{E}[(V \bullet S)_N]$.

(c) Calculate the distribution of $(V \bullet S)_N$ and use this to compute $\mathbb{E}[(V \bullet S)_N]$ again.

(d) Can you find a gambling system $V$ with $(V \bullet S)_N = {S_N}^4$?

If you have feedback regarding the exercise sheets, please send a mail to Jakob Heiss.