Algebraic Topology II FS 2020
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Exercise Sheet 4

Orientations

1. Show that deleting a point from a manifold of dimension greater than 1 does not affect orientability of the manifold.

2. Recall that given a topological space X, a *covering space* of X consists of a topological space Y and a continuous map $p\colon Y \to X$ such that for each point $x \in X$ there is an open neighborhoud U of x in X such that $p^{-1}(U)$ is a union of disjoint open sets each of which is mapped homeomorphically onto U by p. Show that every covering space of an orientable manifold is an orientable manifold.

3. For a map $f\colon M \to N$ between closed orientable n-manifolds with fundamental classes $[M]$ and $[N]$, the *degree* of f is defined to be the integer d such that $f_*([M]) = d[N]$, so the sign of the degree depends on the choice of fundamental classes. Show that for any connected closed orientable n-manifold M there is a map $f : M \to S^n$ of degree 1. (*)

4. Show that for a connected non-orientable manifold M there is a unique orientable double cover $\widetilde{M}$ of M. *Hint:* Assume that there is a second orientable double cover $\overline{M}$ of M and construct an isomorphism between $\overline{M}$ and $\widetilde{M}$.