Now we show:

Thm IV. 3. A symmetric space of non-compact type is \((AT(\mathbb{R}))\).

Recall from Remark IV. 14:

Prop. IV. 14 Let \(M\) be SS of non-compact type with associated \(\mathbb{R}\) \((G, \mathfrak{g})\) and assume \(M\) is endowed with the \(Is(M)^{-}\) inner metric coming from restriction to \(\mathfrak{g}\) of the Killing form. Then \(\exp: T_p M \rightarrow M\) is a distance increasing diffeo.

Observe that this implies that \(M\) is uniquely geodesic.
Lemma II.5 Let

\[ \Delta = \]

\[ \overrightarrow{p} \]

and

\[ \overrightarrow{\Delta} = \]

\[ \overrightarrow{q} \]

comparison triangle. Then:

\[ a \leq \bar{a}, \quad p \leq \bar{p}, \quad r \leq \bar{r}. \]

Proof: Let thus \( p, q, r \in M \) and three geodesic segments \( [p, q], [p, r], [r, q] \) in \( M \). Then \( [p, q] \) and...
\[ \overline{IV}.14 \]

\([p, r]\) are determined by unit tangent vectors \(v_q, v_r\) in \(T_p M\).

With \(a = d(p, q)\), \(b = d(p, r)\)

we have then:

\[ \text{Exp}_p (a v_q) = q, \text{Exp}_p (b v_r) = r. \]

By definition \(\lambda = \text{angle at } 0 \text{ between } v_q \text{ and } v_r.\)

Let

\[ \text{Exp}_p \]
be the comparison triangle. Then:
\[ a^2 + b^2 - 2ab \cos \alpha = c^2 = d(q, r)^2 \]
\[ \geq \frac{1}{2} ab - b^2 \leq c^2 + b^2 - 2ab \cos \alpha \]
which implies:
\[ \cos \alpha \leq \frac{c}{a} \]
But since \( \alpha, \beta \) are in \([0, \pi]\) we get:
\[ \alpha \geq \beta. \]

Proof of Theorem 5.3.

Proceeds in two steps:

1. Let \( \Delta(p, q, r) \subset M \)

\[ \Delta(p, q, r) \subset \mathbb{E}^2 \]
Claim: \( d(p, x_0) \geq d(\overline{p}, \overline{x}) \).

For \( \Delta(p, x_1, x_2) \) and \( \Delta(p, x_1, x_2, \overline{x}) \), take the corresponding comparison triangles in \( \mathbb{E}^2 \):

And observe by Lemma 1.5 that:

\[
\delta_0 \geq \delta, \quad \delta_0' \geq \delta', \quad \delta_0 + \delta_0' \geq \pi.
\]

Let \( p_0, x_0, r_0 \) be colinear such that

\[
d(x_0, r_0) = d(\overline{x}_0, \overline{r}_0) \quad \text{and} \quad \delta_0'' = \delta(p, x_0, r_0).\]
Since \( \beta' \geq \beta_0 \) we conclude from the cosine theorem that \( d(p_0, r) \geq d(p_0, r') \).

This implies:

\[
d(\overline{p}, \overline{r}) = d(p_0, r) \geq d(p_0, r') = d(\overline{p}, \overline{r}').
\]

Since \( d(\overline{r}, r_0) = d(\overline{r}, \overline{p}) \), \( d(\overline{r}, r_0') = d(\overline{r}, \overline{r}) \)

the cosine theorem implies

\[
\overline{p} \geq \beta_0.
\]

and hence: \( d(\overline{p}, \overline{r}) \geq d(\overline{r}, \overline{r}_0) = d(\overline{r}, r') \).
(2) Now we complete the proof.

Let \( \Delta_0 \) be the comparison triangle of \( \Delta \). Then by (11): \( \Delta (p, x) \leq \Delta (p_0, x_0) \).

Next let

\[ \Delta' = \Delta' (p_0, x_0, y_0) \text{ be the comparison triangle of } \Delta (p, x, y) : \]
Then \(d(p_0', x_0) = d(p, x_1) \leq d(p_0, x_0)\)

Since \(d(q_0, p_0) = d(q_0', p_0')\) and
\(d(q_0, x_0) = d(q_0', x_1')\)
the cosine law implies \(\beta' \leq \beta\).

Thus:
\[d(y, x) \leq d(y_0', x_0) \leq d(q_0, x_0)\]

Claim (1)
Then we obtain:

**Thm I. 6.**

Let $H$ be a $S^2$ of non-compact type and $IS(H) < G < IS(M)$. Let $K = Stab(G)$.

1. $K$ meets every connected component of $G$. In particular, $IS(M)$ is of finite index in $IS(M)$.

2. Any compact subgroup $V < G$ has a fixed point in $M$.

3. The set $\{ Stab(p) : p \in M \}$ coincides with the set of maximal subgroups of $G$. 
Proof:

(1) \( G/K_0 \) is a \( \mathbb{R} \)-covering of \( M = G/K \) with \( \mathbb{R} \)-cover group \( K/K_0 \).

Since \( M \) is diffeomorphic to \( T \times M \), in particular simply connected, we must have that \( G/K_0 \) is diffeomorphic to \( M \times K/K_0 \), which implies the first assertion.

(2) Let \( S = U^{\infty} \). Then \( S \) is bounded and \( U \)-invariant and Prop I.2 (i) implies that \( S \subseteq M \) is \( U \)-fixed.

(3) The fact that any compact subgroup of \( G \) is contained in some \( \text{Stab}(\rho) \) follows from (2). The fact that if \( p \neq q \)

\( \text{Stab}(p) \neq \text{Stab}(q) \) requires an additional
argument which we sketch here:

If for some $p \neq q$, $\frac{\text{Stab}(p)}{6} = \frac{\text{Stab}(q)}{6}$, then there is $z \in \mathbb{P}$, $z \not\in \text{Stab}(y)$ with $\text{deg}(y) z = 0$. Thus every irreducible factor of $(y, z)$ would have the same property, contradicting irreducibility. \[\Box\]