# Percolation Theory - Exercise Sheet 6

NCCR SwissMAP - Master Class in Mathematical Physics
Spring term 2020 (401-4604-20L)

Throughout this exercise sheet, we consider percolation on $(\mathbb{Z}^2, E)$.

**Exercise 6.1.** Let $p = 1/2$. Define

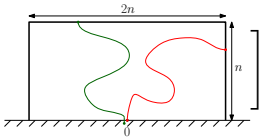
$$\pi(m, n) := \mathbb{P}_{\frac{1}{2}}[\Lambda_m \longleftrightarrow \partial\Lambda_n]$$

for $m \leq n$. Prove that there exists a constant $c > 0$ such that for all $n_3 \geq n_2 \geq 2n_1$

$$c\,\pi(n_1, n_2)\pi(n_2, n_3) \leq \pi(n_1, n_3) \leq \pi(n_1, n_2)\pi(n_2, n_3).$$

**Exercise 6.2.** Prove that there exists $c, C > 0$ such that for all $n \geq 1$,

$$c \cdot \frac{1}{n} \leq \mathbb{P}_{\frac{1}{2}}\Big[ \quad \Big] \leq C \cdot \frac{1}{n}.$$




**Exercise 6.3.** Prove that there exists $c, C > 0$ such that for all $n \geq 1$,

$$c \cdot \frac{1}{n^2} \leq \mathbb{P}_{\frac{1}{2}}\Big[ \quad \Big] \leq C \cdot \frac{1}{n^2}.$$