Multiple Choice 4.1 True or false? Motivate your answer.

Consider the Cauchy problem

$$\begin{cases} y' = \sqrt{|y|} & \text{for } t > 0, \\ y(0) = 0, \end{cases}$$

You notice that that $y \equiv 0$ solves the problem. Consequently, without any further computation, you can say that there exists a sufficiently small half-interval $I = [0, \varepsilon)$, $\varepsilon > 0$, where 0 is the only solution.

☐ True ☐ False

Multiple Choice 4.2 Choose the correct statement. Motivate your answer.

Consider for $n \geq 2$ a function

$$f : (a, b) \to \mathbb{R}^n, \qquad f(t) = (f_1(t), \dots, f_n(t)).$$

In order for f to be differentiable:

(a) it is *necessary*, but in general not sufficient, that each f_j is differentiable. ☐

(b) it is *sufficient*, but in general not necessary, that each f_j is differentiable. ☐

(c) it is *necessary and sufficient* that each f_j's is differentiable. ☐

Exercise 4.1 Find the general solution of the ODE:

$$y^{(4)} + 2y'' + y = f(x),$$

when

(a) $f(x) = \sin x$,

(b) $f(x) = \mathrm{e}^{2x}$,

(c) $f(x) = \sin x + \mathrm{e}^{2x}$.

Exercise 4.2 Solve the following ODE with the method of the variation of constants:

$$y'' + 4y = \frac{1}{\sin(2x)}.$$

Exercise 4.3 Solve the following ODE/Cauchy problems. If you apply the method of separation of variables, be careful not to divide by zero!

(a) $y' - y = \sin x$,

(b) $\begin{cases} y' = (x+y)^2, & \text{for } x \in (-\pi/2, \pi/2), \\ y(0) = 1. \end{cases}$

(c) $\begin{cases} y' = \sqrt{\dfrac{1-y^2}{1-x^2}}, & \text{for } x \in (-\pi/2, \pi/2), \\ y(0) = 0. \end{cases}$

(d) $yy' - (1+y)x^2 = 0$,

Note for (d): you will not be able to write explicitly every solution (this often happens when dealing with nonconstant coefficient ODEs). Is suffices that you find an implicit relation for y that does not involve its derivatives.