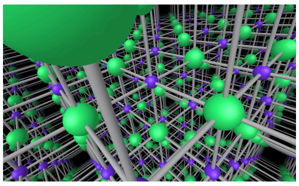
GEOMETRIE!

You boil it in sawdust: you salt it in glue:
You condense it with locusts and tape:
Still keeping one principal object in view—
To preserve its symmetrical shape.

–*Lewis Carroll*



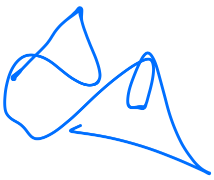
NaCl

- sehr symmetrisch

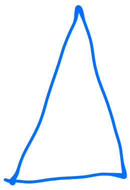
Die Symmetrie:

- Wie symmetrisch ist eine Figur?
- Wie drückt man das aus? Mit welchen mathematischen Strukturen?

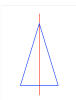
→ Gruppen (von Isometrien)



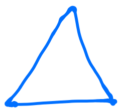
asymmetrisch



symmetrisc



symmetrisch

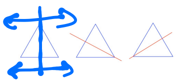


regulärer Dreieck

noch symmetrischer

Die Identität

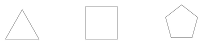
= 3 Drehungen
(0°, 120°, 240°)



= 3 Spiegelungen

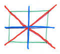
= 6 Symmetrien des gleichseitigen Dreiecks.

Abschnitt 5 Symmetrien von n-Ecken



reguläre 3-Eck, 4-Eck, 5-Eck

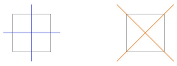
4-Eck:



0, 90, 180, 270

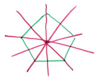
4 reflection plus 4 rotations

8 Symmetrien



2 verschiedene Arte von Spiegelungen

5-Eck



nur 1 Art Spiegelung

Def. 1) Sei F eine Figur in $\mathbb{R}^n$. Eine <u>Symmetrie</u> von F ist eine Isometrie

$$\phi : \mathbb{R}^n \to \mathbb{R}^n$$

die F erhält:

$$\phi(F) = F$$

2)

$$Sym(F) := \{ \phi : \mathbb{R}^n \to \mathbb{R}^n \mid \phi(F) = F \}$$

$$= \{ \text{Symmetrien von } F \}$$

Def. Diedergruppe

$$D_n := Sym(P_n)$$

wobei

$$P_n := \text{reguläres } n\text{-Eck}$$

Frage: Verifiziere, dass D_n $2n$ Symmetrien enthält:

- n Spiegelungen
- 1 Identitätsabbildung
- $n-1$ eigentliche Drehungen,

$$F_1 \subseteq F_2 \iff (x \in F_1 \Rightarrow x \in F_2)$$

$$O(n) = Sym(S^{n-1}), \quad S^{n-1} \subseteq \mathbb{R}^n.$$

3 Metadata

Tom Ilmanen, lecturer

Raphael Appenzeller, organizer

Lectures: Tuesday 16-18 weekly:

15.09.; 22.09.; 29.09.; 06.10.; 13.10.; 20.10.; 27.10.; 03.11.; 10.11.; 17.11.; 24.11.; 01.12.; 08.12.; 15.12 (exam).

Exercise classes: 21.09.; 05.10.; 19.10.; 02.11.; 16.11.; 30.11.; 14.12. (biweekly)

Website: `https://metaphor.ethz.ch/x/2020/hs/401-1511-00L`

Exercises: `https://metaphor.ethz.ch/x/2020/hs/401-1511-00L/ex`

Forum: `https://forum.math.ethz.ch/t/geometrie-herbst-2020/277`

Exam: 15.12.20 in class.

Texts:

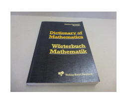
1) H. Knörrer, *Geometrie*, `https://www.springer.com/de/book/9783322939807`. The course corresponds roughly to Chapter 1, pp. 1-64.

2) For group theory: D. Saracino, *Abstract Algebra: A First Course*, pp 1-132, `https://www.waveland.com/browse.php?t=483`. It looks friendly.

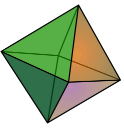
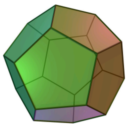
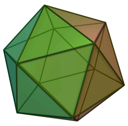
3) A lovely introduction to crystallography (goes beyond the class, but easy reading) is M. Senechal, *Crystalline Symmetries, An informal mathematical introduction*, 1990.

4) Additional reference books include Rotman, Brieskorn I, III, Burns-Glazer, Fischer, Jänich, and others. See Chapter 18 for more books, software, etc.

5) Mathematical dictionary: G. Eisenreich, R. Sube, *Dictionary of Mathematics; Wörterbuch Mathematik*, Verlag Harry Deutsch, 1987.

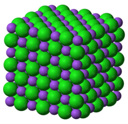


Images: Details of the picture credits are in Section 69.

Viel Symmetrien.

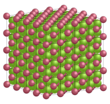
Frage: Wieviel?



Salz
$NaCl$

∞ - viel Symmetrien

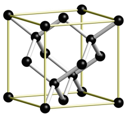
http://www.geometrygames.org/CrystalFlight/index.html



Fluorit
CaF_2

∞ - viel

Frage: Haben $NaCl$ und CaF_2 die gleichen Symmetrien?



Diamant
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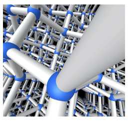
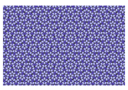
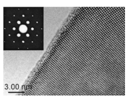
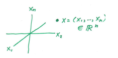




2 penrose transparencies.



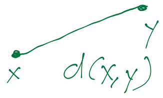


] $x \in \mathbb{R}^n$ heisst:
x ist Element von $\mathbb{R}^n$

$\mathbb{R}^n = \{$alle n-Tupel
$x = (x_1, x_2, \dots, x_n)$:
$x_1, \dots, x_n \in \underline{\underline{\mathbb{R}}}\}$

reelle Zahlen

$\mathbb{R}^2$ = Ebene
$\mathbb{R}^3$ = Raum

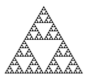


$$d(x,y) := \sqrt{(x_1-y_1)^2 + \dots + (x_n-y_n)^2}$$

wobei $x = (x_1, \dots, x_n)$
$y = (y_1, \dots, y_n)$

$$\begin{array}{rcl} d: \mathbb{R}^n \times \mathbb{R}^n & \longrightarrow & [0,\infty) := \{t \in \mathbb{R} : 0 \le t < \infty\} \\ \Psi & & \Psi \\ (x,y) & \longmapsto & d(x,y) \end{array}$$

d heisst die <u>Euklidische Metrik</u> auf $\mathbb{R}^n$


The Sierpinski triangle

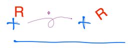
Def: Eine <u>Figur</u> ist irgendeine Teilmenge von $\mathbb{R}^n$.

Def. Eine <u>Isometrie</u> oder <u>Bewegung</u> von $\mathbb{R}^n$ ist eine surjektive Funktion

$$\begin{array}{rcl} \phi: \mathbb{R}^n & \longrightarrow & \mathbb{R}^n, \\ \Psi & & \Psi \\ x & \longmapsto & \phi(x) \end{array}$$

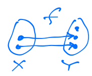
die den Abstand zwischen Punkten erhält:

$$d(\phi(x), \phi(y)) = d(x,y)$$

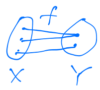


Sei $f: X \to Y$ Abbildung
(d.h. Funktion)

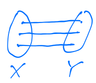
injectiv:



surjektiv:



bijektiv:



.... Schluss:
Isometrien sind:
<u>bijektiv</u>.

Beispiele von Isometrien:

- Die Identität $id_{\mathbb{R}^n}: \mathbb{R}^n \to \mathbb{R}^n$
  $x \mapsto x$
- Drehungen
- Translationen (Verschiebungen)
- Spiegelungen in Ebenen, Geraden, oder Punkten

Exotischer:

- Drehspiegelungen
- Gleitspiegelungen
- Schraubbewegungen