

Probability Theory

Exercise Sheet 6

Exercise 6.1 Let $(X_i)_{i \geq 1}$ be i.i.d. with symmetric stable distribution of parameter $\alpha \in (0, 2)$, see lecture notes p. 63.

- (a) Find the distribution of $n^{-1/\alpha}(X_1 + \dots + X_n)$.
- (b) Does $\frac{1}{\sqrt{n}}(X_1 + \dots + X_n)$ converge in distribution?

Exercise 6.2 Let X_1, X_2, \dots be independent random variables for which there exists a constant $M > 0$, such that $|X_n| \leq M$, P -a.s. for $n = 1, 2, \dots$. We write $S_n = X_1 + \dots + X_n$. Show that, if $\sum \text{Var}(X_n) = \infty$, then there exist constants a_n, b_n such that $(S_n - b_n)/a_n$ converges in distribution towards a standard normal random variable.

Hint: Use the Lindeberg-Feller theorem (Theorem 2.24, p. 71 of the lecture notes).

Exercise 6.3 Show that when $Y_k, k \geq 1$ are independent uniformly bounded random variables such that $\sum_k Y_k$ converges P -a.s., then $\sum_k \text{Var}(Y_k) < \infty$.

Hint: consider independent copies $\tilde{Y}_k, k \geq 1$ of the $Y_k, k \geq 1$ and use Exercise 6.2 with $X_k = Y_k - \tilde{Y}_k, k \geq 1$.

Submission: until 12:00, Nov. 3, through the webpage of the course. You should carefully follow the **submission instructions** on the webpage to get your solutions back.

Office hours: See the webpage for detailed information

- Präsenz (Group 3): Mon. and Thu., 12:00-13:00 in HG G32.6. with previous reservation.
- Probability Theory Assistants: Tue. 15:30-16:30 and Wed. 11:00-12:00 via Zoom with a 10 minutes slot reservation.

Exercise class: Online. In-person exercise classes need previous registration each week.

Exercise sheets and further information are also available on:
<https://metaphor.ethz.ch/x/2020/hs/401-3601-00L/>