Non-Life Insurance: Mathematics and Statistics

Exercise sheet 4

Exercise 4.1 Poisson Model and Negative-Binomial Model
Suppose that we are given the following claim count data of ten years:

t	1	2	3	4	5	6	7	8	9	10
N_t	1'000	997	985	989	1'056	1'070	994	986	1'093	1'054
v_t	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000

Table 1: Observed claim counts N_t and corresponding volumes v_t.

(a) Estimate the claim frequency parameter $\lambda > 0$ of the Poisson model. Moreover, calculate a prediction interval which should contain roughly 70% of the observed claim frequencies N_t/v_t. What do you observe?

(b) Perform a χ^2-goodness-of-fit test at significance level of 5% to test the null hypothesis of having Poisson distributions.

(c) Estimate the claim frequency parameter $\lambda > 0$ and the dispersion parameter $\gamma > 0$ of the negative-binomial model. Moreover, calculate a prediction interval which should contain roughly 70% of the observed claim frequencies N_t/v_t. What do you observe?

Exercise 4.2 χ^2-Goodness-of-Fit-Analysis (R Exercise)
In this exercise we analyze the sensitivity of the χ^2-goodness-of-fit test (of having a Poisson distribution as claim count distribution) in situations where the claim counts are simulated from a Poisson distribution and a negative binomial distribution, respectively.

(a) Write an R code that generates $n = 10'000$ times claim counts $N_1, \ldots, N_T \overset{\text{i.i.d.}}{\sim} \text{Poi}(\lambda v)$ with $T = 10$, $\lambda = 10\%$ and $v = 10'000$. Apply for each of these n replications of $N_1, \ldots, N_T$ a χ^2-goodness-of-fit test at significance level of 5% of having a Poisson distribution as claim count distribution. Answer the following questions:

   (i) What can you say about the distribution of the n MLEs of λ?

   (ii) Consider a QQ plot to analyze whether the n values of the test statistic may indeed come from a χ^2-distribution with $T - 1 = 9$ degrees of freedom.

   (iii) How often do we wrongly reject the null hypothesis H_0 of having a Poisson distribution as claim count distribution?

(b) Write an R code that generates $n = 10'000$ times claim counts $N_1, \ldots, N_T \overset{\text{i.i.d.}}{\sim} \text{NegBin}(\lambda v, \gamma)$ with $T = 10$, $\lambda = 10\%$, $v = 10'000$ and $\gamma \in \{100, 1'000, 10'000\}$. Apply for each of these n replications of $N_1, \ldots, N_T$ a χ^2-goodness-of-fit test at significance level of 5% of having a Poisson distribution as claim count distribution. Answer the following questions:

   (i) How often are we able to reject the null hypothesis H_0 of having a Poisson distribution as claim count distribution?

   (ii) Does the size of γ influence this percentage?

Exercise 4.3 Claim Count Distribution
Suppose that in a given line of business of an insurance company the numbers of claims of the last ten years are modeled by random variables $N_1, \ldots, N_{10}$. We assume that $N_1, \ldots, N_{10}$ are i.i.d. and that we have collected the following observations:

t	1	2	3	4	5	6	7	8	9	10
N_t	7	21	19	18	25	17	33	6	39	28

Table 2: Observed numbers of claims N_t over the last ten years.

Which claim count distribution would you prefer in this situation? Give a short argument.

Exercise 4.4 Method of Moments
The i.i.d. claim sizes $Y_1, \ldots, Y_8$ are supposed to follow a Gamma distribution with unknown shape parameter $\gamma > 0$ and unknown scale parameter $c > 0$. We assume that we have the following observations for $Y_1, \ldots, Y_8$:

$$y_1 = 7, \quad y_2 = 8, \quad y_3 = 10, \quad y_4 = 9, \quad y_5 = 5, \quad y_6 = 11, \quad y_7 = 6, \quad y_8 = 8.$$

Use the method of moments to estimate the parameters γ and c.