Non-Life Insurance: Mathematics and Statistics

Exercise sheet 7

Exercise 7.1 Re-Insurance Covers and Leverage Effect
In Figure 1 we compare the distribution function of a loss $Y \sim \Gamma(1, \frac{1}{400})$ (in black color) to the distribution function of the loss after applying different re-insurance covers to Y (in red color).

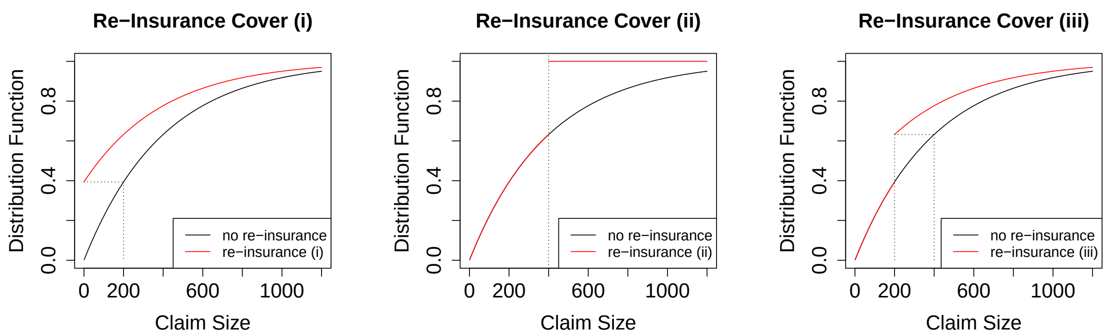

Figure 1: Distribution functions implied by re-insurance contracts (i), (ii) and (iii).

(a) Let $d > 0$. Show that Y satisfies

$$\mathbb{E}[(Y-d)_+] = \mathbb{P}[Y > d]\mathbb{E}[Y].$$

(b) Can you explicitly determine the re-insurance covers from the graphs in Figure 1?

(c) Calculate the expected values of these modified contracts.

(d) Assume that a first claim Y_0 has the same distribution as Y, and that a second claim Y_1 fulfills $Y_1 \stackrel{(d)}{=} (1+i)Y_0$, for a constant inflation rate $i > 0$. Let $d > 0$. Show the leverage effect

$$\mathbb{E}[(Y_1-d)_+] > (1+i)\mathbb{E}[(Y_0-d)_+].$$

Give an appropriate explanation for this leverage effect.

Exercise 7.2 Inflation and Deductible
We assume that this year's claims in a storm insurance portfolio have been modeled by a Pareto distribution with threshold $\theta > 0$ and tail index $\alpha > 1$. The threshold θ can be understood as deductible. Suppose that the inflation in the next year is expected to be $100 \cdot r\,\%$ for some $r > 0$. By how much do we have to increase the deductible θ next year such that the average claim payment remains unchanged?

Exercise 7.3 Normal Approximation
Assume that the total claim amount S has a compound Poisson distribution with expected number of claims $\lambda v = 1'000$ and claim sizes following a gamma distribution with shape parameter $\gamma = 100$ and scale parameter $c = \frac{1}{10}$. Use the normal approximation to estimate the 0.95-quantile $q_{0.95}$ and the 0.99-quantile $q_{0.99}$ of S.

Exercise 7.4 Translated Gamma and Translated Log-Normal Approximation
Consider the same setup as in Exercise 7.3. This time we use the translated gamma and the translated log-normal approximation to estimate the quantiles $q_{0.95}$ and $q_{0.99}$ of the total claim amount S.

(a) Use the translated gamma approximation to estimate $q_{0.95}$ and $q_{0.99}$.

(b) Use the translated log-normal approximation to estimate $q_{0.95}$ and $q_{0.99}$.

(c) Compare the results of (a) and (b) to the results found in Exercise 7.3, where we used the normal approximation.