Non-Life Insurance: Mathematics and Statistics

Exercise sheet 9

Exercise 9.1 Utility Indifference Price
In this exercise we calculate the premium for the accident insurance of a given company COMP using the utility indifference price principle. We suppose that all employees of COMP have been divided into two groups, depending on their work, and that the total claim amounts S_1 and S_2 of the two groups are independent and compound Poisson distributed with volumes, claim frequencies and claim size distributions as given in Table 1.

Group i	v_i	λ_i	$Y_1^{(i)}$
1	2'000	50%	$\Gamma(\gamma = 20, c = 0.01)$
2	10'000	10%	$\text{expo}(\kappa = 0.005)$

Table 1: Volumes, claim frequencies and claim size distributions for the two groups of employees.

We write $S = S_1 + S_2$ for the total claim amount of COMP. Let c_0 be the initial capital of the insurance company that sells accident insurance to COMP.

(a) Let u be a risk-averse utility function. Show that if the utility indifference price $\pi = \pi(u, S, c_0)$ exists, then it is unique and satisfies $\pi > \mathbb{E}[S]$.

(b) Calculate $\mathbb{E}[S]$.

(c) Calculate π using the utility indifference price principle for the exponential utility function with parameter $\alpha = 1.5 \cdot 10^{-6}$.

(d) What happens to π if we replace the compound Poisson distributions of S_1 and S_2 by Gaussian distributions with the same corresponding first two moments?

(e) For this part we assume that S has a general compound Poisson distribution with expected number of claims $\lambda v \in \mathbb{N}$ and i.i.d. claim sizes $(Y_i)_{i \geq 1}$ for which the moment generating function M_{Y_1} exists at α for a given $\alpha > 0$. Moreover, let u be the exponential utility function with parameter α, and $c_0 > 0$ a given initial capital. We write $\pi = \pi(u, S, c_0)$ for the utility indifference price for S. Now define

$$\widetilde{S} = \sum_{i=1}^{\lambda v} Y_i,$$

i.e. for $\widetilde{S}$ the number of claims is exactly given by λv. Calculate the utility indifference price $\widetilde{\pi} = \widetilde{\pi}(u, \widetilde{S}, c_0)$ for $\widetilde{S}$ and compare $\widetilde{\pi}$ to π.

Exercise 9.2 Value-at-Risk and Expected Shortfall
Suppose that for the yearly claim amount S of an insurance company in a given line of business we have $S \sim \text{LN}(\mu, \sigma^2)$ with $\mu = 20$ and $\sigma^2 = 0.015$. Moreover, we set the cost-of-capital rate $r_{\text{CoC}} = 6\%$. Then, the premium π_{CoC} for the considered line of business using the cost-of-capital pricing principle with risk measure ρ is given by

$$\pi_{\text{CoC}} = \mathbb{E}[S] + r_{\text{CoC}} \cdot \rho(S - \mathbb{E}[S]).$$

(a) Calculate π_{CoC} using the value-at-risk (VaR) risk measure at security level $1-q=99.5\%$.

(b) Calculate π_{CoC} using the expected shortfall risk measure at security level $1-q=99\%$.

(c) Which security level is needed such that π_{CoC} using the VaR risk measure is equal to the price calculated in (b)?

(d) Let U and V be two independent copies of $\log S$. Show that on the one hand

$$\mathrm{VaR}_{1-q}(U+V) \; > \; \mathrm{VaR}_{1-q}(U)+\mathrm{VaR}_{1-q}(V)$$

for all $1-q \in \left(0, \frac{1}{2}\right)$, but on the other hand

$$\mathrm{VaR}_{1-q}(U+V) \; < \; \mathrm{VaR}_{1-q}(U)+\mathrm{VaR}_{1-q}(V)$$

for all $1-q \in \left(\frac{1}{2}, 1\right)$. In particular, the VaR is not subadditive, and hence not coherent.

Exercise 9.3 Variance Loading Principle

We would like to insure the car fleet given in Table 2 under the assumption that the total claim amounts for passenger cars, delivery vans and trucks can be modeled by independent compound Poisson distributions.

i	v_i	λ_i	$\mathbb{E}[Y_1^{(i)}]$	$\mathrm{Vco}(Y_1^{(i)})$
passenger car	40	25%	2'000	2.5
delivery van	30	23%	1'700	2.0
truck	10	19%	4'000	3.0

Table 2: Volumes, claim frequencies, expected claim sizes and coefficients of variation of the claim sizes for the three sections of the car fleet.

(a) Calculate the expected claim amount of the car fleet.

(b) Calculate the premium for the car fleet using the variance loading principle with $\alpha = 3 \cdot 10^{-6}$.

Exercise 9.4 Esscher Premium

Let S be a random variable with distribution function F and moment generating function M_S. Assume that there exists $r_0 > 0$ such that $M_S(r) < \infty$ for all $r \in (-r_0, r_0)$. For $\alpha \in (0, r_0)$, let π_α denote the Esscher premium of S.

(a) Show that if S is non-deterministic, then π_α is strictly increasing in α.

(b) Show that the Esscher premium for small values of α boils down to a variance loading principle.

(c) Suppose that $S \sim \mathrm{CompPoi}(\lambda v, G)$, where $\lambda v > 0$ and G is the distribution function of a gamma distribution with shape parameter $\gamma > 0$ and scale parameter $c > 0$. For which values of α does π_α exist? Calculate π_α where it is defined.