Mathematical Finance

Exercise sheet 7

Exercise 7.1 Show that in finite discrete time we have

$$(NA) \implies (NUPBR).$$

Exercise 7.2

(a) Construct an example where (NA) holds but not $(NUPBR)$.

(b) Construct an example where $(NUPBR)$ holds but not (NA).

Exercise 7.3 Let X be a Banach space and let Y be a closed point-separating linear subset of the dual space. Show that X, together with the topology making all linear functionals of Y continuous (the $\sigma(X,Y)$ topology) is metrizable if and only if X is finite dimensional.

Exercise 7.4 Let $C \subseteq X$ be a convex subset of X, a Banach space. Show that C is closed in X if and only if it is closed with respect to the weak topology $\sigma(X, X^*)$.

Exercise 7.5 Consider the Bachelier model, taking $[0,1]$ as the time interval and where the price of the risky asset is given by

$$S_t = \int_0^t \sigma dB_s.$$

Consider $\mathcal{X}_1 := \{\vartheta \bullet S, \vartheta \in \Theta^1_{adm}\}$, the set of wealth processes produced by admissible strategies.

(a) Show that $\mathcal{X}_1$ has the concatenation property: for any bounded, predictable $H, G \geq 0$ with $HG = 0$ and for any $X, Y \in \mathcal{X}_1$, if

$$Z = (H \bullet X) + (G \bullet Y) \geq -1$$

then $Z \in \mathcal{X}_1$.

(b) Show that $\mathcal{X}_1$ is closed in the Emery topology.

Exercise 7.6 (Python) Let B be a standard Brownian motion motion and consider a market consisting of three assets $S^0 \equiv 1$, $S^1_t = \exp(B_t)$ and $S^2_t = \exp\left(\frac{1}{2}B_t\right)$, $t \in [0,T]$, for some $0 < T < \infty$. Verify numerically that the market admits scalable arbitrage.