

# Mathematics of Machine Learning

## Homework 10

Instructors: Afonso S. Bandeira & Nikita Zhivotovskiy  
Course Coordinator: Pedro Abdalla Teixeira

May 07, 2021

Try to solve the questions before looking to the answers. Every item must be proved rigorously. Starred problems are harder.

### Problem 1

Consider i.i.d standard Gaussian random variable  $g_1, \dots, g_n$  and a unit vector  $a = (a_1, \dots, a_n)$ .

- (a) Use the rotation invariance property of the Gaussian random variable to derive a concentration inequality for the sum  $\sum_{i=1}^n a_i g_i$ .
- (b) Use the Chernoff Method to derive a similar concentration inequality for the sum  $\sum_{i=1}^n a_i g_i$ . Compare with the inequality in letter "a".

## Problem 2

Show that the Sauer-Shelah lemma is tight, i.e, give an example of a family of events  $\mathcal{A}$  with VC-dimension equal to  $d$  such that the shatter function is  $S_{\mathcal{A}}(n) = \sum_{i=1}^d \binom{n}{i}$ .

## Problem 3

Compute the VC-dimension of the following classes

- (a) Indicator function of sets of the form  $[a, b] \cup [c, d]$  in  $\mathbb{R}$ .
- (b) Indicator function of all circles in  $\mathbb{R}^2$ .
- (c) All sets of the form  $\{x \in \mathbb{R} : \sin(xt) \geq 0\}$  for all  $t > 0$ .