

Mathematics of Machine Learning

Homework 5

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March 26, 2021

Try to solve the questions before looking to the answers. Every item must be proved rigorously. Starred problems are harder.

Problem 1

Given m and d the smallest worst case coherence is the minimum worst case coherence among all frames of m unit norm vectors in d dimensions. Can you find the smallest worst case coherence for the following cases:

- (a) 2 vectors in 2 dimensions ($d = 2, m = 2$)
- (b) 3 vectors in 2 dimensions ($d = 2, m = 3$)

Problem 2

For a matrix Φ , prove that $\text{spark}(\Phi) \leq \text{rank}(\Phi) + 1$

Problem 3

Consider a tight frame $\{\phi_1, \dots, \phi_m\} \in \mathbb{C}^d$ with frame bound A .

- (a) Prove that the associated frame operator S is equal to $c(A)I_d$, where $c(A)$ is a constant that depends only on A and I_d is the $d \times d$ identity matrix.
- (b) Prove that if $A = 1$ and all vectors have unit norm, then the frame must be an orthonormal basis.
- (c) Can you give an example of a tight frame that is not an orthogonal basis?