11.1. Wrapping up Lecture 21

1. Show $\dfrac{\partial w}{\partial \nu} = \dfrac{\partial v}{\partial x_n} \circ T^{-1}$ and change variables.
2. Have a look to the proof of point (iv) of Lemma 2 in lecture 21 and don't forget to use the variance-minimizing property of the mean-value that was discussed at the end of the lecture.

11.2. Divergence theorem reloaded

By definition of $w \in H_0^1(B_r)$, there exists a sequence $(w_k)_{k\in\mathbb{N}}$ in $C_c^\infty(B_r)$ such that $\|w - w_k\|_{H^1(B_r)} \to 0$ as $k \to \infty$.

11.3. Everything you need to know on iteration lemmas

First part: warm-up. Exploit monotonicity of f and iterate the hypothesis.

Second part: refinements.

(a) Choose $\tau \in \,]0,1[$ such that $2A\tau^\alpha = \tau^\gamma$.

(b) Part (a) is the base case of the induction.

(c) Exploit monotonicity of f and part (b). A geometric sum is bounded.

11.4. Interpolation inequality (Lemma 10.4.2)

Towards a contradiction, suppose there exists a sequence $(x_k)_{k\in\mathbb{N}}$ in X and some $\varepsilon_0 > 0$ such that $1 = \|x_k\|_Y \geq \varepsilon_0 \|x_k\|_X + k\|x_k\|_Z$ for every $k \in \mathbb{N}$.

11.5. Abstract method of continuity

(a) Recall Satz 6.2.2.

(b) If $t_0 \in I$, then A_{t_0} is in fact bijective by assumption $(*)$ and $A_{t_0}^{-1} \in L(Y,X)$. Using Satz 2.2.7, show that if $t \in [0,1]$ is sufficiently close to t_0, then A_t is bijective.

(c) Consider a sequence $(t_k)_{k\in\mathbb{N}}$ in I such that $t_k \to t_\infty$ as $k \to \infty$ for some $t_\infty \in [0,1]$. Exploit surjectivity of A_{t_k} and the inequality from assumption $(*)$.