7.1. Completeness of Campanato spaces

Notice that a Cauchy sequence $(u_k)_{k\in\mathbb{N}}$ in $(\mathcal{L}^{p,\lambda}(\Omega), \|\cdot\|_{\mathcal{L}^{p,\lambda}(\Omega)})$ is a Cauchy sequence in $(L^p(\Omega), \|\cdot\|_{L^p(\Omega)})$ and therefore has a limit $v \in L^p(\Omega)$. Prove that $(u_m - (u_m)_{x_0,r})$ converges to $(v - v_{x_0,r})$ in $L^p(\Omega \cap B_r(x_0))$ as $m \to \infty$.

7.2. Vanishing weak gradient

For sufficiently small $\varepsilon > 0$ restrict u to the domain $\Omega_\varepsilon := \{x \in \Omega \mid \operatorname{dist}(x, \partial\Omega) > \varepsilon\}$ and mollify. Argue that the mollification u_ε is constant in Ω_ε and prove that these constants (which may depend on ε) converge as $\varepsilon \to 0$.

7.3. Hölder continuity of functions in $W^{2,n}$

Reformulate the proof of Satz 9.1.1. for this special case.

7.4. Uniform bounds on functions in $W^{n,1}$

Modify the proof of Satz 8.5.1.

7.5. A variant of the Poincaré inequality

You can prove this statement by direct computation or (more elegantly) by contradiction. Argue and exploit that the embedding $W^{1,p}(\Omega) \hookrightarrow L^p(\Omega)$ is compact. The result of Problem 7.2 might be helpful.

7.6. Explosion of the Poincaré constant

Find a suitable function $u \in W^{1,p}(\Omega_k)$ whose gradient is supported in the thin part A_k of the domain Ω_k.

Need more hints?
Come to office hours!