Introduction to Mathematical Finance
Dylan Possamaï

Assignment 3

1. **Call option properties**

   Let $C_t(T,K;S)$ be the price at some time $t\geq 0$ of an European Call option with strike $K\geq 0$, maturity $T\geq 0$ and underlying asset with value $(S_t)_{t\in[0,T]}$.

   1) Prove that for any $(T_1,T_2,t,K)\in[0,+\infty)\times[T_1,+\infty)\times[0,T_1]\times[0,+\infty)$, we have

   $$C_t(T_1,KB(T_1,T_2);S)\leq C_t(T_2,K;S).$$

   In particular, show that the map $T\longmapsto C_t(T,K;S)$ is non–decreasing when interest rates are non–negative.

   2) Prove that for any $(T,t,K_1,K_2)\in[0,+\infty)\times[0,T]\times[0,+\infty)\times[0,+\infty)$, we have

   $$\big|C_t(T,K_1;S)-C_t(T,K_2;S)\big|\leq B(t,T)\big|K_2-K_1\big|.$$

   Deduce that the map $K\longmapsto C_t(T,K;S)$ is continuous and Lebesgue–almost everywhere differentiable.

   3) Prove that we have for any $(T,t)\in[0,+\infty)\times[0,T]$

   $$\lim_{K\to+\infty}C_t(T,K;S)=0.$$

2. **Put options properties**

   Prove Proposition 1.4.19 from the lecture notes.