

Assignment 4

1. Call-Call and Call-Put options

Consider a financial market without arbitrages. We consider a Call-Call option, that is to say an option whose underlying is itself an option. More precisely, a Call-Call option with strike K_1 and maturity T_1 on a Call option with strike K_2 and maturity $T_2 > T_1$ and underlying S has the following payoff at T_1

$$CC_{T_1}(K_1, T_1; K_2, T_2, S) := (C_{T_1}(T_2, K_2; S) - K_1)^+.$$

Similarly, we define Put-Call option with strike K_1 and maturity T_1 on a Call option with strike K_2 and maturity $T_2 > T_1$ and underlying S has having the following payoff at T_1

$$PP_{T_1}(K_1, T_1; K_2, T_2, S) := (K_1 - C_{T_1}(T_2, K_2; S))^+.$$

Prove the following parity relationship for any $t \in [0, T_1]$

$$CC_t(K_1, T_1; K_2, T_2, S) - PP_t(K_1, T_1; K_2, T_2, S) = C_t(T_2, K_2, S) - K_1 B(t, T_1).$$

2. Bull Spread

A Bull Spread consist in buying a call with strike K_1 and selling a call with strike $K_2 > K_1$.

- 1) Does this strategy have an initial cost?
- 2) Calculate and draw the corresponding payoff.
- 3) What do you think could be the purpose of this strategy?

3. Bottom Straddle

A Bottom Straddle consists in buying a call and a put with the same strike and the same maturity. Same questions as in the previous exercise.

4. Butterfly Spread

A Butterfly Spread consists in selling 2 calls with strike K_2 and buying a Call with strike K_1 and a call with strike K_3 , with $0 \leq K_2 - K_1 = K_3 - K_2$. Same questions as in the previous exercise.

5. Barrier option

We consider a Up and Out Call i.e. an option with payoff

$$(S_T - K)^+ \mathbf{1}_{\{\sup_{0 \leq t \leq T} S_t \leq L\}}.$$

- 1) Explain the name 'barrier option'.
- 2) What happens when $L \leq K$ or $S_0 > L$? We assume now that $L > K$ and $S_0 < L$.
- 3) Show that if the price of this option is above $\frac{L-K}{L} S_0$, there exists an arbitrage opportunity.