# Lecture 10 - Prewritten definitions and theorems

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Rem 8.7:

Thm 8.8: If $g \in \pi_1(G, Y, T)$ has a reduced expression different from 1, then $g \neq 1$. In particular, the vertex groups $G_v$, $v \in Y^0$ are canonically embedded in $\pi_1(G, Y, T)$.

**Def 8.9:** Let $p: X \to Y$ be a morphism from a tree $X$ to a connected graph $Y$ and let $T$ be a spanning tree of $Y$. A pair $(\tilde{T}, \tilde{Y})$ of subtrees in $X$ is called a <u>lift of</u> $(T, Y)$ if $\tilde{T} \subseteq \tilde{Y}$ and

i) each edge from $\tilde{Y}^1 \setminus \tilde{T}^1$ has initial or terminal vertex in $\tilde{T}$;

ii) $p|_{\tilde{T}}: \tilde{T} \to T$ is an isomorphism and $p$ maps $\tilde{Y}^1 \setminus \tilde{T}^1$ bijectively onto $Y^1 \setminus T^1$.

If $v \in Y^0 = T^0$, we write $\tilde{v}$ for its preimage in $\tilde{T}^0$ and for $e \in Y^1$, we write $\tilde{e}$ for its preimage in $\tilde{Y}^1$.

**Thm 8.10** (Structure theorem of Bass-Serre theory I):
Let $(G, Y)$ be a graph of groups, $T \subseteq Y$ a spanning tree and $G = \pi_1(G, Y, T)$. The group $G$ acts without inversions on on a tree $X$ such that the factor graph $G \backslash X$ is isomorphic to $Y$ and such that the stabilisers of the vertices and edges of $X$ are conjugate to the (canonically embedded) subgroups $G_v$, $v \in Y^0$, and $G_e$, $e \in Y^1$.

Moreover, for the corresponding projection $p: X \to Y = G \backslash X$ there is a lift $(\tilde{T}, \tilde{Y})$ of $(T, Y)$ such that

i) for $\tilde{v} \in \tilde{T}^0$, $\mathrm{Stab}_G(\tilde{v}) = G_v$;

ii) for $\tilde{e} \in \tilde{Y}^1$ with $\alpha(\tilde{e}) \in \tilde{T}^0$, $\mathrm{Stab}_G(\tilde{e}) = \alpha_e(G_e)$;

iii) if $\tilde{e} \in \tilde{Y}^1$ with $\omega(\tilde{e}) \notin \tilde{T}^0$, then $t_e^{-1}(\omega(\tilde{e})) \in \tilde{T}^0$.

Cor 8.10: Every finite subgroup of $\pi_1(G, Y, T)$ can be conjugated into a vertex group.

Thm 8.11 (Structure theorem II): Let $G$ be a group that acts without inversion on a tree $X$. Then there is a graph of groups $(G, Y)$ s. t. $G \cong \pi_1(G, Y, T)$ for some (any) spanning tree $T$ of $Y$.