Lecture 11 - Prewritten definitions and theorems

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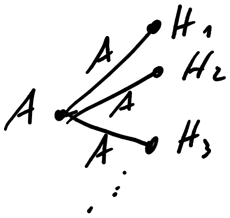
Cor 8.11: Every finite subgroup of $\pi_1(G, Y, T)$ can be conjugated into a vertex group.

Thm 8.12 (Structure theorem II): Let G be a group that acts without inversion on a tree X. Then there is a graph of groups (G, Y) s.t. $G \cong \pi_1(G, Y, T)$ for some (any) spanning tree T of Y.

9. Applications of Bass-Serre Theory I: Subgroups of amalgamated products

Example 9.1:

Thm 9.2 (Kurosh): Let $H = \underset{A}{*}\{H_i \mid i \in I\}$, i.e. $H = \pi_1(G, Y, Y)$, where (G, Y) is the graph of groups given by



("free product of $\{H_i\}_{i \in I}$ amalgamated over A").

Let $G \leq H$ s.t. $G \cap xAx^{-1} = \{1\}$ for all $x \in H$.
Then there exists a free group F and a system of representatives X_i of double cosets $G\backslash H/H_i$ such that

$$G \cong F * \underset{\substack{i \in I \\ x \in X_i}}{*'} G \cap xH_ix^{-1}).$$

Rem 9.3:

Cor 9.4: Let $G = A * B$ and $g, h \in G$ with $[g,h]=1$. Then g and h are either contained in the same conjugate of A or B or there is $x \in G$ with $g, h \in \langle x \rangle$ (i.e. $\langle g, h \rangle$ cyclic).