Lecture 13 - Prewritten definitions and theorems

Dienstag, 18. Mai 2021 15:44

Lemma 10.6: Let $e \in X^1$ and $k \in \mathbb{N}$.
There are only finitely many k-separators that contain e.

Pf.: We prove the statement by induction on k.
$k = 1$ is trivial.

Def 10.7:

- A cut is a set of vertices $C \subseteq X^0$ such that δC is finite and C and C^c both contain a ray.

- If there is a cut, define
$$\kappa := \min_{C \text{ cut}} \{ |\delta C| \}.$$

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- A cut C with $|\delta C| = \kappa$ is called <u>thin</u>.

Lemma 10.8: Let C and D be thin cuts. If $C \cap D$ and $C^c \cap D^c$ are cuts, then they are thin.

Def 10.9: Let $C, D \subseteq X^{\circ}$.

- The intersections

$$C \cap D, \quad C \cap D^c, \quad C^c \cap D, \quad C^c \cap D^c$$

are called the <u>corners</u> of C and D. We say that $C \cap D$ is <u>opposite</u> to $C^c \cap D^c$ and $C \cap D^c$ is opposite to $C^c \cap D^c$.

- C and D are <u>nested</u> if one of their corners is empty.
- <u>C is constant on D</u> if either $D \subset C$ or $C \cap D = \emptyset$.

Lem. 10.10: i) there are two opposite corners of C and D such that is not nested with either of them, then E is is neither nested with C nor with D.

ii) Let C and D be cuts that are not nested. If a set E is nested with C and D, then E is nested with all corners of C and D.

Def 10.11: Let C be a cut. We denote by $M(C)$ the set of thin cuts that are not nested with C.
We set $m(C) := |M(C)|$.

Lem. 10.12: Let C and D be thin cuts that are not nested. If $C \cap D$ and $C^c \cap D^c$ are cuts, then
$$m(C \cap D) + m(C^c \cap D^c) < m(C) + m(D) \,. \quad (*)$$

Def 10.13: Let $\mathcal{C}$ be the set of all thin cuts and set
$$m := \min_{C \in \mathcal{C}} \{ m(C) \} .$$

A thin cut C with $m(C) = m$ is called <u>optimally nested</u>.

Thm. 10.14: Optimally nested cuts are all nested with each other.