Lecture 14 - Prewritten definitions and theorems

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10.b Trees from cuts

Def 10.15: Let X be a set. A binary relation $\leq$ on X is called a partial order if it satisfies the following:

i) $x \leq x \quad \forall x \in X$ (reflexivity)

ii) $[x \leq y \wedge y \leq x] \Rightarrow x = y$ (antisymmetry)

iii) $[x \leq y \wedge y \leq z] \Rightarrow x \leq z$ (transitivity)

Def 10.16: Let X be a graph. If $e, f \in X^1$, we write $e \leq f$ if there is a reduced path $e = e_1, e_2, \ldots, e_n = f$.

Observation 10.17: If X is a tree, then $\leq$ determines a partial order on X^1. In addition, the following conditions are satisfied:

i) if $e \leq f$, then $\bar{f} \leq \bar{e}$;

ii) if $e \leq f$, there are only finite many $d \in X^1$ for which one has $e \leq d \leq f$;

iii) for any pair e, f, at least one of
$$e \leq f, \quad e \leq \bar{f}, \quad \bar{e} \leq f, \quad \bar{e} \leq f$$
holds.

iv) for no pair e, f, we have $e \leq f$ and $e \leq \bar{f}$;

v) for no pair e, f, we have $e \leq f$ and $\bar{e} \leq f$.

Thm 10.18: Let $(E, \leq)$ be a partially ordered set with a map $E \to E$, $e \mapsto \bar{e}$ s.t. $\bar{\bar{e}} = e$ and suppose that conditions i) - v) from Obs. 10.17 are satisfied.
Then there is a tree T with $E = T^{\wedge}$ and the order relation on E is precisely the one defined in Def 10.16.

Thm 10.4 (Stallings): A finitely generated group G has more than one end if and only if it splits over some finite subgroup.