Lecture 8 - Prewritten definitions and theorems

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Cor. 7.4: Let $G^* = \langle G, t \mid t^{-1} a t = \varphi(a), a \in A \rangle$ be an HNN-extension with associated subgroups $A \xrightarrow{\varphi} B$. Then the canonical projection $c: G * \langle t \rangle \to G^*$ restricts to isomorphisms on G and $\langle t \rangle$. After identifying these groups with their images, A and B are conjugate in G^* under t.

Cor 7.5: Let H be a group with subgroups G and $\langle t \rangle \cong \mathbb{Z}$, let $A, B \leq H$ such that $t^{-1} A t = B$. If every element of H has a unique normal form, then $H \cong G^*$.

Prop 7.6: Let $H, K \leq G$ and $\varphi: H \to K$ an isomorphism. Then the HNN-extension $G^* = \langle G, t \mid t^{-1} h t = \varphi(h) \rangle$ can be seen as a subgroup of an amalgamated product.

Thm 7.7 (HNN '49): Every countable group C can be embedded in a group G that is generated by two elements of infinite order and such that:

i) G has n-torsion iff C has n-torsion

ii) If C is finitely presented, so is G.

Cor. 7.8 (B. Neumann): There are $2^{\aleph_0}$ many non-isomorphic 2-generator groups.

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Thm 7.9 (HNN '49): Every countable group can be embedded into a group where all elements of the same order are conjugate.

Thm 7.10: Every countable group G can be embedded into a countable, simple, divisible group.

Cor 7.11: There are $2^{\aleph_0}$ countable simple groups.

Def 7.12: A loop is a graph consisting of one vertex and two mutually inverse edges:

Rem 7.13:

Thm 7.14: Let $G = H^{*}_{\varphi} = \langle H, t \mid t^{-1} a t = \varphi(a)\ \forall a \in A \rangle$ be an HNN-extension. Then there is a tree X s.t. G acts on X without inversions and such that the quotient graph $G \backslash X$ is a loop. Moreover, there is a segment

$$\tilde{Y} = \underset{\alpha(e)}{\bullet} \xrightarrow{\ e\ } \underset{\omega(e)}{\bullet}$$

in X s.t. $G_e = A$, $G_{\alpha(e)} = H$ and $G_{\omega(e)} = tHt^{-1}$.

Thm 7.15: Let G be a group that acts without inversions on a tree X s.t. the quotient $G \backslash X$ is a loop and let $Y = \bullet \xrightarrow{e} \bullet$ be a lift of $G \backslash X$.
Let $g \in G$ s.t. $g(\alpha(e)) = \omega(e)$. Let $\varphi : G_e \to g^{-1} G_e g$ be the isomorphism given by conjugation with g. Then $g^{-1} G_e g \leq G_{\alpha(e)}$ and the homomorphism

[illegible] monomorphism given by conjugation with g s.t.
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$$\langle G_{\alpha(e)}, t \mid t^{-1} a t = \varphi(a) \ \forall a \in G_e \rangle \to G$$

given by sending t to g is an isomorphism. I.e.
G can be written as an HNN-extension $G \cong G_{\alpha(e)}^{*}$.