11.1. Eigenvalues of the rectangle For each of the following 5 statement you have to establish whether it is true or false.

Insert your answers in the following grid. Write clearly **T** if the statement is true and **F** if the statement is false. We will accept also **R** if the statement is *richtig* (which is the German word for *true*).

Only the answers in the grid will be taken into consideration for grading.

Question	1	2	3	4	5
Answer					

Let $R := (0,a) \times (0,b)$ for $a,b > 0$. Let $\lambda_1 \leq \lambda_2 \leq \cdots$ be the eigenvalues (with multiplicity) of $-\Delta$ with Dirichlet boundary conditions on R, namely the values of $\lambda \in \mathbb{R}$ such that the following problem has a nontrivial solution

$$\begin{cases} -\Delta u = \lambda u & \text{in } R, \\ u = 0 & \text{on } \partial R. \end{cases}$$

1. There exists a negative eigenvalue.
2. If $a = 2\pi$ and $b = 5\pi$, then $\lambda_1 = \frac{29}{100}$.
3. If $a = b = \pi$, the multiplicity of 65 as eigenvalue is 2.
4. If $a = 5\pi, b = 2021$, there is not an *integer* eigenvalue.
5. If $a = b = 1$, then $\lambda_{2021} \leq 100$.

$$Av = \lambda v$$
$$\det(\lambda I - A) = 0$$