1.1. Linear ODE with constant coefficients. Solve (i.e. determine the set of all solutions of) the following differential equations for $y(x)$:

(a) $y'' - \omega^2 y = 0$,

(b) $y'' + \omega^2 y = 0$,

(c) $y'' + 3y' + 4y = \cos(2x)$.

1.2. First-order ODE with variable coefficients. Solve (i.e. determine the set of all solutions of) the following differential equations for $y(x)$:

(a) $y' - x^2 y = 0,\ x \in \mathbb{R}$,

(b) $y' - y/x = x,\ x > 0$,

(c) $y' + x^5 y = x^6 + 1,\ x \in \mathbb{R}$,

(d) $y' = (x + y)^2$,

(e) $y' - y = \sin x$,

(f) $y\,y' - (1 + y)x^2 = 0$.

Tips: ODE of 1st order may be solved by *separation of variables* or by substitution. For (c), multiply the equation with $\mathrm{e}^{f(x)}$, where f is a suitable function. For (f), y will not explicitly be a function of x. It is enough to write a relation between the function y and the variable x that does not contain any derivatives of y.

1.3. Initial and boundary value problems. Solve the following Cauchy problems:

(a) $$\begin{cases} y' &= 2\mathrm{e}^{2x} \quad \forall x \in \mathbb{R}, \\ y(0) &= 2. \end{cases}$$

(b) $$\begin{cases} y''(x) + 4y(x) &= 0 \quad \forall x \in (0, L)\ (L > 0 \text{ given}), \\ y(0) &= 0, \\ y(L) &= 2. \end{cases}$$

1.4. Spring pendulum A spring pendulum consists of a coil spring and a mass test piece (with mass m) attached to it, which can move in a straight line in the direction in which the spring extends or retracts. Let $K > 0$ be the spring constant and $\omega^2 := K/m$, then the equation of motion of the spring pendulum is given by

$$\ddot{x}(t) + \omega^2 x(t) = 0. \tag{1}$$

Find the solution of the differential equation (1):

(a) with the initial conditions $x(0) = 1, \dot{x}(0) = 2\omega$.

(b) with the boundary conditions $x(0) = 1, x(\frac{\pi}{2\omega}) = 1$.

1.5. Classification of PDEs I. Suppose a, b, f and g are differentiable functions. Tell whether the following differential equations in $u(x, y)$ are linear and homogeneous, linear and inhomogeneous, or non-linear and (in any case) tell their order. For every linear differential equation of 2nd order, tell whether the equation is elliptic, hyperbolic or parabolic.

(a) $u_{xxx} + u_y = f$

(b) $au_{xx} + bu^2 = 0$

(c) $u_x u_y = 0$

(d) $2u_{xx} + u_x + 2u_{xy} + 2u_{yy} = 0$

(e) $(1 - x^2)u_{xx} - 2xyu_{xy} + (1 - y^2)u_{yy} = g$ in $\Omega = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$.

1.6. Classification of PDEs II.

Suppose a, b and g differentiable functions with $g > 0$.

Tell whether the following differential equations in $u(x, y)$ are linear and homogeneous, linear and inhomogeneous, or non-linear and tell their order. For every linear differential equation of 2nd order, tell whether the equation is elliptic, hyperbolic or parabolic.

(a) $au_{xxx} + b(u^4 + u) = 0$

(b) $a^2u_{xx} + u_x u_y = 1$

(c) $4u_{xx} + u_x + u_{xy} + 6u_{yy} = 0$

(d) $(x^2 - 2)u_{xx} + 4xyu_{xy} + (y^2 - 2)u_{yy} = g$ in $\Omega = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 16\}$.

1.7. Dreaming a Cauchy-Lipschitz theorem for the wave equation. (*Achtung: for this problem we don't expect you to write down anything; this is instead about thinking ahead.*) Read (the first three pages of) **Lesson 16** in Farlow's textbook, about the 'derivation' of the (one-dimensional) wave equation, as describing a vibrating string. Compare this equation with Newton's equation, as recalled in class. What 'data' would you expect one has to specify for such wave equation for something like the Cauchy-Lipschitz theorem (i.e. local existence and uniqueness) to hold true? We will discuss this at length in the coming lectures.