11.1. Eigenvalues of the rectangle For each of the following 5 statement you have to establish whether it is true or false.

Insert your answers in the following grid. Write clearly **T** if the statement is true and **F** if the statement is false. We will accept also **R** if the statement is *richtig* (which is the German word for *true*).

Only the answers in the grid will be taken into consideration for grading.

Question	1	2	3	4	5
Answer					

Let $R := (0,a) \times (0,b)$ for $a, b > 0$. Let $\lambda_1 \leq \lambda_2 \leq \cdots$ be the eigenvalues (with multiplicity) of $-\Delta$ with Dirichlet boundary conditions on R, namely the values of $\lambda \in \mathbb{R}$ such that the following problem has a nontrivial solution

$$\begin{cases} -\Delta u = \lambda u & \text{in } R, \\ u = 0 & \text{on } \partial R. \end{cases}$$

1. There exists a negative eigenvalue.
2. If $a = 2\pi$ and $b = 5\pi$, then $\lambda_1 = \frac{29}{100}$.
3. If $a = b = \pi$, the multiplicity of 65 as eigenvalue is 2.
4. If $a = 5\pi, b = 2021$, there is not an *integer* eigenvalue.
5. If $a = b = 1$, then $\lambda_{2021} \leq 100$.

11.2. Laplace equation in the square Let $R := (0,1) \times (0,1) \subset \mathbb{R}^2$. Compute the solution $u : R \to \mathbb{R}$ of the following Dirichlet problem

$$\begin{cases} \Delta u(x,y) = 0 & \text{in } R \\ u(x,y) = f(x,y) & \text{on } \partial R, \end{cases}$$

where

(a)

$$f(x,y) = \begin{cases} 0 & \text{for} \quad y = 0\,, \\ 0 & \text{for} \quad y = 1\,, \\ 0 & \text{for} \quad x = 0\,, \\ -\sin(2\pi y)\cos(2\pi y) & \text{for} \quad x = 1. \end{cases}$$

(b)

$$f(x,y) = \begin{cases} x(x-1) & \text{for} \quad y = 0\,, \\ 0 & \text{for} \quad y = 1\,, \\ 0 & \text{for} \quad x = 0\,, \\ 0 & \text{for} \quad x = 1. \end{cases}$$

Hint: Write u as a sum of functions of the form $X_n(x)Y_n(y)$.

11.3. Laplace equation with mixed boundary conditions Compute the solution $u : R \to \mathbb{R}$ of the following boundary value problem

$$\begin{cases} \Delta u(x,y) = 0 & \text{for } (x,y) \in (0,1)^2, \\ u(x,y) = 0 & \text{for } (x,y) \in \{0,1\} \times (0,1) \cup (0,1) \times \{0\}, \\ u + u_y = \sin(\pi x) & \text{for } (x,y) \in (0,1) \times \{1\}. \end{cases}$$