3.1. Heat equation. Solve the following IBVP:

$$
\begin{aligned}
u_t - u_{xx} &= 0 && x \in (0,\pi)\,,\ t > 0\,,\\
u(0,t) &= 0 && t > 0\,,\\
u(\pi,t) &= 0 && t > 0\,,\\
u(x,0) &= \begin{cases} 1 & \text{if } \frac{\pi}{3} \le x \le \frac{2\pi}{3}\,,\\ 0 & \text{if } x < \frac{\pi}{3} \text{ or } \frac{2\pi}{3} < x\,. \end{cases}
\end{aligned}
$$

3.2. Extreme points of piecewise C^1. Given $T > 0$, let $f : (-T, T) \to \mathbb{R}$ be a function such that there exists a partition $\{t_0 = -T < t_1 < \ldots < t_k = T\}$ and a constant $C > 0$ such that $f_{|(t_i,t_{i+1})}$ is a C^1 function and $|f(t)| + |f'(t)| \le C$ for all $t_i < t < t_{i+1}$, for $i = 0, 1, \ldots, k-1$.

Prove that, for any $i = 0, 1, \ldots, k-1$, there exists $f^+(t_i) := \lim_{t \to t_i^+} f(t)$ as well as $f^-(t_i) := \lim_{t \to t_i^-} f(t)$.

3.3. Fourier series for symmetric functions. Let f be a 2π-periodic function. Prove the following statements:

(a) If f is even, i.e. $f(-t) = f(t)\ \forall t$, then the real Fourier series of f has the following form

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kt)\,.$$

(b) If f is odd, i.e. $f(-t) = -f(t)\ \forall t$, then the real Fourier series of f has the following form

$$\sum_{k=1}^{\infty} b_k \sin(kt)\,.$$

3.4. Fourier series I. Compute the real Fourier series (sine/cosine form) of the 2-periodic function

$$f(x) = 1 - x^2, \quad -1 < x < 1\,.$$

3.5. Convergent series. Compute the value of the following series

$$\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)^3}\,.$$

Hint: Compute the Fourier series of 2π-periodic function $f(x) = x^3 - \pi^2 x$ for $x \in (-\pi, \pi)$.