9.1. Homogeneous wave equation on the real line. Let $u : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ be a solution of the PDE

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{for } x \in \mathbb{R},\ t > 0, \\ u(x,0) = e^{-x^2} & \text{for } x \in \mathbb{R}, \\ u_t(x,0) = 0 & \text{for } x \in \mathbb{R}. \end{cases}$$

1. Find the PDE satisfied by the Fourier transform of u with respect to the x variable.
2. Solve the ODE satisfied by $t \mapsto \hat{u}(\xi, t)$ when $\xi \in \mathbb{R}$ is fixed.
3. By computing the inverse Fourier transform of $\hat{u}$, find an explicit formula for u.

9.2. Inhomogeneous wave equation on the real line. Let $u : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ be a solution of the PDE

$$\begin{cases} u_{tt} - 4u_{xx} = \sin(4t) + x & \text{for } x \in \mathbb{R},\ t > 0, \\ u(x,0) = 2x^2 & \text{for } x \in \mathbb{R}, \\ u_t(x,0) = 6\cos(x) & \text{for } x \in \mathbb{R}. \end{cases}$$

1. Find a particular solution v of the equation (which does not necessarily satisfy the initial conditions) by employing the Ansatz $v(x,t) = v_1(x) + v_2(t)$.
2. Write down the PDE satisfied by $w := u - v$.
3. Use d'Alembert's formula to find an explicit formula for w and deduce an explicit formula for u.