Exam Geometrie HS 2021

Exercise 1 **[4 points]**

Without justification, decide whether the following statements are true or false.

1. Let γ be a locally length-minimizing curve in a metric space. Then every subcurve of γ also locally minimizes lengths.
2. If (X, d) is a metric space, then (X, d^2) is also a metric space.
3. If the respective angles of two spherical triangles are equal, then their areas are equal.
4. The transformation $f\colon \hat{\mathbb{C}} \to \hat{\mathbb{C}}, f(z) = (\overline{z}+1)/(\overline{z}-1)$ preserves orientation.

1 point for every correct answer, 0 points for every unanswered question, and -1 points for every wrong answer. The minimum number of points for this exercise is 0.

Exercise 2 **[4 points]**

Let K be a circle in $\mathbb{C}$. Let E be a cline in $\hat{\mathbb{C}}$ that intersects K in two points at a $90°$-angle. Show that inversion in K sends E to itself.

Exercise 3 **[4 points]**

Let $\mathbb{H}^2$ be the Poincaré disk. Let C be the circle with midpoint $1 + i \in \mathbb{C}$ and radius 1. Let α be the hyperbolic geodesic $C \cap \mathbb{H}^2$. Find the hyperbolic distance from 0 to the nearest point of α.

Exercise 4 **[4 points]**

Let $\mathbb{H}^2$ be the Poincaré disk. Find an isometry of $\mathbb{H}^2$ that sends the point $-\frac{1}{2} \in \mathbb{H}^2$ to $+\frac{1}{2} \in \mathbb{H}^2$, but does not preserve the origin.

Exercise 5 **[4 points]**

Recall that $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ is the extended complex plane. Let $\hat{\sigma}\colon S^2 \to \hat{\mathbb{C}}$ be the ordinary stereographic projection, based at the north pole $N = (0, 0, 1)$. Let $\hat{\rho}\colon S^2 \to \hat{\mathbb{C}}$ be the stereographic projection based at the *south* pole $S = (0, 0, -1)$. Find an explicit formula for the map $\hat{\sigma} \circ \hat{\rho}^{-1}$.