

Ising Model: Exercise sheet 10

Exercise 1. Let $\Lambda \subset \mathbb{Z}^d$ be a finite set. Let $\Delta \subset \Lambda$. Let f be an increasing function on $(\sigma_x)_{x \in \Delta}$. Using the domain Markov property and the comparison between boundary conditions, prove that

$$\langle f \rangle_{\Lambda}^+ \leq \langle f \rangle_{\Delta}^+.$$

Exercise 2. Let $n \geq 1$. We say that the box 0 is connected to Λ_n for the configuration σ if there exists a path of vertices with + spin from 0 to $\partial\Lambda_n$. We denote this event $\{0 \leftrightarrow \partial\Lambda_n\}$. Prove that

$$\langle \sigma_0 | 0 \not\leftrightarrow \partial\Lambda_n \rangle_{\Lambda_n}^+ \leq 0.$$

Exercise 3. Let $\beta > 0$. Let $\omega \in \{-1, 1\}^{\{0,1\}}$. For $\sigma \in \{-1, 1\}^{\{0,1\}}$, we define

$$H^\omega(\sigma) = -\beta\sigma_0\sigma_1 - 10\beta\sigma_0\omega_0 - 10\beta\sigma_1\omega_1.$$

Give all the boundary conditions ω such that we have

$$\langle \sigma_0\sigma_1 \rangle^\omega \geq \langle \sigma_0 \rangle^\omega \langle \sigma_1 \rangle^\omega.$$