# Ising Model: Exercise sheet 8

*Exercise* 1. ($\star$) For $x \in \mathbb{Z}^d$, we define

$$\langle \sigma_0 \sigma_x \rangle_\beta^+ = \lim_{n \to \infty} \langle \sigma_0 \sigma_x \rangle_{\Lambda_n, \beta}^+. \tag{1}$$

The aim of this exercise is to prove that the following quantity is finite for $\beta < \beta_c$

$$\chi(\beta) := \sum_{x \in \mathbb{Z}^d} \langle \sigma_0 \sigma_x \rangle_\beta^+.$$

1. Prove that the limit in (1) is well-defined.
2. Prove that for $\beta < \beta_c$, there exists $c > 0$ such that for any $x \in \mathbb{Z}^d$ we have $\langle \sigma_0 \sigma_x \rangle_\beta^+ \leq e^{-c\|x\|}$.
3. Conclude that $\chi(\beta)$ is finite.

*Exercise* 2. Let $G = (V, E)$ be a finite graph and $J$ be some nonnegative weights on the edges. Let $M, N$ be two independent ppp$(J)$ on $G$. Let $x, y, z, t \in V$ be four distinct vertices. Prove that

$$\mathbb{P}[\partial M = xy, \partial N = zt, x \overset{M+N}{\not\leftrightarrow} z] \leq \mathbb{P}[\partial M = xy, \partial N = \emptyset, x \overset{M+N}{\not\leftrightarrow} z].$$

*Hint: condition on the cluster of $x$ in $M + N$*