Non-Life Insurance: Mathematics and Statistics

Exercise sheet 8

Exercise 8.1 Panjer Algorithm
In this exercise we use the Panjer algorithm to calculate monthly health insurance premiums for different franchises d. We assume that the yearly claim amount

$$S = \sum_{i=1}^{N} Y_i$$

of a given customer is compound Poisson distributed with $N \sim \text{Poi}(1)$ and $Y_1 \overset{(d)}{=} k + Z$, where $k = 100$ CHF and $Z \sim \text{LN}(\mu = 7.8, \sigma^2 = 1)$. In health insurance the policyholder can choose between different franchises $d \in \{300, 500, 1'000, 1'500, 2'000, 2'500\}$. The franchise d describes the threshold up to which the policyholder has to pay everything by himself. Moreover, the policyholder has to pay $\alpha = 10\%$ of the part of the total claim amount S that exceeds the franchise d, but only up to a maximal amount of $M = 700$ CHF. Thus, the yearly amount paid by the customer is given by

$$S_{\text{ins}} = \min\{S, d\} + \min\{\alpha \cdot (S - d)_+, M\}.$$

If we define $\pi_0 = \mathbb{E}[S]$ and $\pi_{\text{ins}} = \mathbb{E}[S_{\text{ins}}]$, the monthly pure risk premium π is given by

$$\pi = \frac{\pi_0 - \pi_{\text{ins}}}{12}.$$

Calculate π for the different franchises $d \in \{300, 500, 1'000, 1'500, 2'000, 2'500\}$ using the Panjer algorithm. In order to apply the Panjer algorithm, discretize the translated log-normal distribution using a span of $s = 10$ and putting all the probability mass to the upper end of the intervals.

Exercise 8.2 Monte Carlo Simulations (R Exercise)
Consider the same setup as in Exercise 7.3. This time we use Monte Carlo simulations to determine the distribution of the total claim amount S.

(a) Write an R code that simulates $n = 100'000$ times the total claim amount S. Compare the resulting distribution function of S to the approximate distribution functions found in Exercises 7.3 and 7.4, where we used the normal, the translated gamma and the translated log-normal approximation.

(b) Write an R code that simulates $n \in \{100, 1'000, 10'000\}$ times the total claim amount S and replicates these simulations 100 times. For each $n \in \{100, 1'000, 10'000\}$ discuss the distribution of the resulting 100 values of the quantiles $q_{0.95}$ and $q_{0.99}$ of S.

Exercise 8.3 Fast Fourier Transform (R Exercise)
Consider the same setup as in Exercise 7.3. Write an R code that applies the fast Fourier transform using a threshold of $n = 2'000'000$ in order to determine the distribution function of the total claim amount S. Compare the resulting distribution function to the distribution function found in Exercise 8.2, where we used Monte Carlo simulations. Moreover, determine the quantiles $q_{0.95}$ and $q_{0.99}$ of S and compare them to the values found in Exercises 7.3 and 7.4, where we used the normal, the translated gamma and the translated log-normal approximation.

Exercise 8.4 Panjer Distribution
Let N be a random variable that has a Panjer distribution with parameters $a, b \in \mathbb{R}$. Calculate $\mathbb{E}[N]$ and $\mathrm{Var}(N)$. What can you say about the ratio of $\mathrm{Var}(N)$ to $\mathbb{E}[N]$?